Chip Design
Summer term 2009

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Exercises 4

1. (a) Show that for quadratic netlength minimization CLIQUE net models can be replaced equivalently by STAR net models by adjusting the netweights. [5 points]

   (b) Conclude that for quadratic netlength minimization it suffices to solve a linear equation system $Ax = b$, where A has $O(|P|+|\mathcal{C}|)$ non-zero entries. [2 points]

2. Let $G = (V, E)$ be a simple graph with $V = \{1, \ldots, n\}$. The Laplacian matrix L_G of G is an $n \times n$-matrix, whose entries $l_{i,j}$, $1 \leq i, j \leq n$, are given by

$$l_{i,j} = \begin{cases} -1 & \text{if } i,j \in E, \\ |\delta(i)| & \text{if } i = j, \textit{and} \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Prove that L_G is positive semidefinite, that is, $x^T L_G x \geq 0$ for all $x \in \mathbb{R}^n$. [2 points]

   (b) Let G be connected and let $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ be the eigenvalues of L_G. Show that $\lambda_1 = 0$ and $\lambda_2 > 0$. [3 points]

   (c) Show that the multiplicity of 0 as an eigenvalue of L_G equals the number of connected components of G. [3 points]

3. Implement a program that computes the minimum area of a rectangular chip image in which a feasible placement exists for a given set of rectangles (without rotations). The program should enumerate all possible sequence pairs and compute longest paths in weighted acyclic digraphs (see Propositions 3.4 and 3.6).

   The implementation must be done either in the C++ or C programming language respecting the C/C++ standard from 1999. You can easily achieve this by using the GNU-compiler (gcc or g++) and by including only standard headers (including the STL).

   The input should be read either from an input pipe or directly from a file. The input format is as follows. The first line contains a number $n \in \mathbb{N}$ specifying the number of rectangles to be placed. The remaining n lines contain two numbers specifying the widths and heights of the rectangles.

Here is an example of an instance with 2 squares with edge lengths 1 and 2:

```
2
1 1
2 2
```

The program should write the result to the standard output. The output must consist of a line with two numbers for width and height of the computed chip area, and a line for each rectangle with two numbers specifying its lower left corner. Assume the lower left corner of the chip area to be $(0, 0)$. An output for the above problem could look like this:

```
2 3
0 0
0 1
```

More sample instances can be found on the webpage of the exercises class. [15 points]

The deadline for submitting solutions for the first and second exercises is May 14 before the lecture (12:15).

The deadline for submitting solutions for the third (programming) exercise by e-mail to `held@or.uni-bonn.de` is May 25.