Chip Design
Summer term 2009

Prof. Dr. Jens Vygen
Dr. Stephan Held

Exercises 2

1. Let $N \subset \mathbb{R}^2$ be a finite set, $(x, y) \in N$ and $x' \in \mathbb{R}$ such that $\{(x'', y'') \in N : x'' < x'\} = \{(x, y)\}$. Show that there exists an optimum rectilinear Steiner tree for N which consists of an optimum rectilinear Steiner tree for $(N \setminus \{(x, y)\}) \cup \{(x', y)\}$ plus the edge $\{(x, y), (x', y)\}$. [2 points]

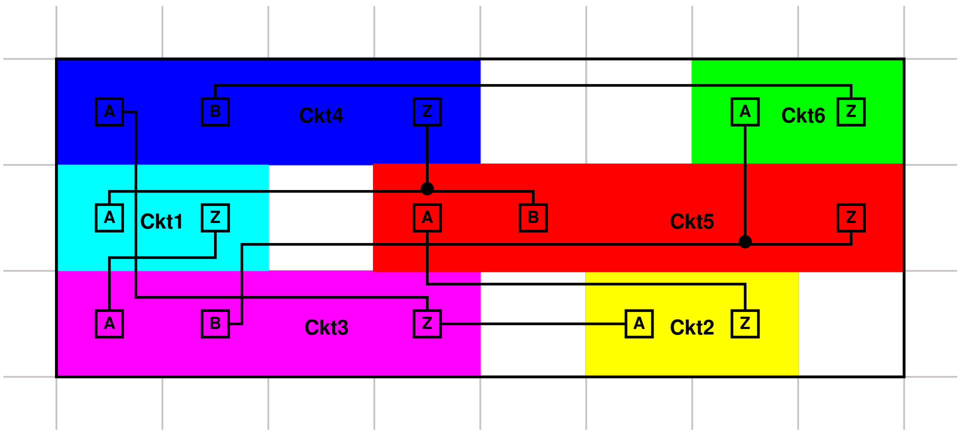
2. Let N be a finite set of pins, and let $\mathcal{D}(p)$ be a set of axis-parallel rectangles for each $p \in N$. We look for an axis-parallel rectangle B with minimum perimeter such that for every $p \in N$ there is a $D \in \mathcal{D}(p)$ with $B \cap D \neq \emptyset$. Let $n := \sum_{p \in N} |\mathcal{D}(p)|$.

   (a) Show that such a rectangle B can be computed in $O(n^4)$ time. [2 points]

   (b) Show that such a rectangle B can be computed in $O(n^3)$ time. [4 points] (Hint: Enumerate possible coordinates for the lower left corner of B.)

3. Consider the placement instance in the figure below. Each of the circuits Ckt1, Ckt2, Ckt3, Ckt4, Ckt5, and Ckt6 must be placed in one of the three circuit rows. Of course, their outlines must also be within the chip area (black outline) and must not intersect each other. The orientation of the circuits must not be changed. The figure shows a legal placement.

   Pins are marked by small squares within their circuits. The centers of these squares are the pin locations; they are fixed relative to their circuit. Nets are described by Steiner trees connecting their pins. These Steiner trees are not pairwise disjoint; therefore Steiner points are drawn as filled circles. Two adjacent parallel grey lines have distance one.



The *Steiner tree netlength* of a placement is defined as

$$\textsc{Steiner}(\mathcal{N}) := \sum_{\mathcal{N}} \textsc{Steiner}(N),$$

where $\mathcal{N}$ is the set of nets in our instance.

Note that, for better readability, not all Steiner trees in the above figure are optimum. The placement in the figure has a Steiner tree netlength of 32.

The *bounding box netlength* of a placement is defined as

$$\mathrm{BB}(\mathcal{N}) := \sum_{N \in \mathcal{N}} \mathrm{BB}(N).$$

(a) What is the maximum possible absolute difference $|\textsc{Steiner}(\mathcal{N}) - \mathrm{BB}(\mathcal{N})|$ for the given instance and all legal placements? [2 points]

(b) Prove that there is no legal placement with $\textsc{Steiner}(\mathcal{N}) < 9$. Can you find a better lower bound? [3 points]

(c) Determine a legal placement of minimum Steiner tree length. [7 points] (If your placement is legal and k units worse than optimum, you will get $\min\{0, 7-k\}$ points.)

The deadline for submitting solutions is April 30 before the lecture (12:15).