Chip Design
Summer term 2009

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Exercises 1

1. Let T be a Steiner tree for a terminal set N in which all leaves are terminals. Prove:

   (a) $|\{v \in V(T) \setminus N : |\delta_T(v)| > 2\}| \le |N| - 2$. [2 points]

   (b) $\sum_{v \in N} (|\delta_T(v)| - 1) = k - 1$, where k is the number of full components in T. [2 points]

2. Let N be an instance of the Rectilinear Steiner Tree Problem and $r \in N$. For a rectilinear Steiner tree T for N we denote by $l(T)$ the maximum length of a path from r to an element of $N \setminus \{r\}$ in T.

   (a) Describe an instance in which no shortest Steiner tree minimizes $l(T)$ and no Steiner tree minimizing $l(T)$ is shortest. [2 points]

   (b) Consider the problem of finding a shortest Steiner tree for which $l(T)$ is minimum among all shortest Steiner trees. Is there always a tree with these properties which is a subgraph of the Hanan grid? [6 points]

3. For a finite set $N \subset \mathbb{R}^2$ we define $\mathrm{BB}(N) := \max_{(x,y)\in N} x - \min_{(x,y)\in N} x + \max_{(x,y)\in N} y - \min_{(x,y)\in N} y$. Moreover, let $\textsc{Steiner}(N)$ be the length of a shortest rectilinear Steiner tree for N, and let $\mathrm{MST}(N)$ be the length of a minimum spanning tree in the complete graph on N, where edge weights are ℓ_1-distances.

   (a) Prove that $\mathrm{BB}(N) \le \textsc{Steiner}(N) \le \mathrm{MST}(N)$ for all finite sets $N \subset \mathbb{R}^2$. [2 points]

   (b) Prove that $\textsc{Steiner}(N) \le \frac{3}{2}\mathrm{BB}(N)$ for all $N \subset \mathbb{R}^2$ with $|N| \le 5$. [4 points]

   (c) Show that there exists no $k \in \mathbb{R}$ with $\textsc{Steiner}(N) \le k \cdot \mathrm{BB}(N)$ for all finite sets $N \subset \mathbb{R}^2$. [2 points]

The deadline for submitting solutions is April 23 before the lecture (12:15).