

## Exercise 11

1. Given a linear time-cost tradeoff instance  $(G, \mathcal{T}, c)$ , show that a solution  $x$  is time-cost optimal if and only if  $C(x) = C_{OPT}(T(x))$ . (5 points)
2. Develop a combinatorial polynomial time algorithm for the (linear) BUDGET-PROBLEM. (5 points)
3. Assume that the cost functions  $c_e, e \in E(G)$ , are strictly decreasing, piecewise linear, and convex functions. Show that the DEADLINE PROBLEM and BUDGET PROBLEM for such cost functions can be solved by oracle algorithms that solve linear instances optimally. (5 points)
4. Given a discrete time-cost tradeoff instance  $(G, \mathcal{T}, c)$  and  $D, B \in \mathbb{R}_{\geq 0}$ , the DISCRETE TIME-COST TRADEOFF DECISION PROBLEM is to decide whether a solution  $x$  with  $T(x) \leq D$  and  $C(x) \leq B$  exists.  
  
Prove that the DISCRETE TIME-COST TRADEOFF DECISION PROBLEM is NP-complete even if  $G$  is a path. (Hint: Recall the KNAPSACK PROBLEM.) (5 points)

The deadline for this exercise is **Tuesday July 8 at 12:15**, before the lecture.