Chip Design
SS 2008

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Exercise 9

1. (a) Show that the worst late slack $\min\{\overline{\text{slack}}(p,\sigma) \mid \sigma \in S(P)\}$ at a pin is at least the minimum worst late slack over its predecessors. (The same holds for the early slacks.) (1 point)

   (b) A signal path P is a path $((v_1,\sigma_1), e_1, (v_2,\sigma_2), e_2, \dots, (v_n,\sigma_n))$, where (v_{i+1},σ_{i+1}) is caused by (v_i,σ_i) by propagation over e_i. The late slack of such a path is defined as

   $$\overline{\text{rat}}(v_n,\sigma_n) - \overline{\text{at}}(v_1,\sigma_1) - \sum_{i=1}^{n-1} \theta_{e_i}(\overline{\text{shape}}(v_i,\sigma_i)).$$

   Assuming delays being invariant in the slews show that the late slack $\overline{\text{slack}}(p,\sigma)$ of a signal $\sigma \in S(p)$ at a node p is the worst late slack of a path through (p,σ).
   (The same holds for the early slacks.) (1 points)

2. (a) Consider a linear optimization problem:

   Problem A:

   $$\begin{aligned} \min\ & c_0 + c_1x_1 + \cdots + c_nx_n \\ s.t.\ & x = (x_1,\dots,x_n) \in D, \end{aligned}$$

   and the deviated problem on the same solution set but with a rational objective function:

   Problem B:

   $$\begin{aligned} \min\ & \frac{a_0 + a_1x_1 + \cdots + a_nx_n}{b_0 + b_1x_1 + \cdots + b_nx_n} \\ s.t.\ & x = (x_1,\dots,x_n) \in D, \end{aligned}$$

   assuming the denominator is always positive. Show that if Problem A is solvable with $O(p(n))$ comparisons and $O(q(n))$ additions, then Problem B is solvable in time $O(p(n)(q(n)+p(n)))$. (Hint: The algorithm is similar to binary search. Assuming knowledge of the optimum value it runs an algorithm for Problem A for verification and performs a test at every comparison to restrict the optimum solution set.) (6 points)

(b) Given a digraph G with costs $c : E(G) \to \mathbb{R}$ and weights $w : E(G) \to \mathbb{R}_{\geq 0}$ such that every cycle $C \subset G$ with negative costs $c(C) := \sum_{e \in E(C)} c(e) < 0$ has positive total weight $w(C) := \sum_{e \in E(C)} w(e) > 0$, show that the minimum ratio $\frac{c(C)}{w(C)}$ of a cycle C with $w(C) > 0$ can be computed in strongly polynomial time using the Bellman-Ford-Moore Algorithm and part (a). (4 points)

(c) Assume that all late slacks in the timing graph are positive. For each net a bound for the maximum feasible detour has to be assigned such that all late slacks remain feasible if the netlength of each net stays within its bounds. As in Problem 4 on Exercise 7 assume the delays being independent from the slew but given as linear functions in the netlength. Show that the maximum possible smallest assigned bound equals the value of a minimum ratio cycle in a slightly modified timing graph. (2 points)

3. A posynomial function $f : \mathbb{R}^n_+ \to \mathbb{R}$ is of the form

$$f(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}, \tag{1}$$

where $K \in \mathbb{N}, c_k > 0$ and $a_{ik} \in \mathbb{R}$.

(a) Give an example for a non-convex posynomial function. (2 points)

(b) Let $\mathbb{R}^n_+ \to \mathbb{R}$ be a posynomial function and $l, u \in \mathbb{R}^n_+, l \leq u$ lower and upper bounds on the variables. Show that each local minimum of f on the box $[l, u]$ is also a global minimum of f on $[l, u]$. (Hint: Transform the problem by logarithmic variable transformation into a convex problem.) (4 points)

The deadline for this exercise is **Tuesday June 24 at 12:15**, before the lecture.