Exercise 3

1. For a finite set \( V \subset \mathbb{R}^2 \), the Voronoï diagram (w.r.t. to the \( l_2 \)-norm) consists of the regions

\[
P_v := \left\{ x \in \mathbb{R}^2 \mid \|x - v\|_2 = \min_{w \in V} \|x - w\|_2 \right\}
\]

for \( v \in V \). The Delaunay triangulation of \( V \) is the graph

\[
(V, \left\{ \{v, w\} \subset V, v \neq w, |P_v \cap P_w| > 1 \right\})
\]

(a) Prove that every minimum spanning tree w.r.t. the \( l_2 \)-norm is a subgraph of the Delaunay triangulation. (4 points)

(b) Does the statement from 1a) hold when using the \( l_1 \)-norm instead of the \( l_2 \)-norm? (2 points)

2. Consider the following algorithm to compute a rectilinear Steiner tree \( T \) for a set \( P \) of points in the plane \( \mathbb{R}^2 \).

1: Choose \( p \in P \) arbitrarily;
2: \( T := (\{p\}, \emptyset), S := P \setminus \{p\} \)
3: while \( S \neq \emptyset \) do
4: Choose \( s \in S \) with minimum \( \text{dist}(s, T) \);
5: Let \( \{u, w\} \in E(T) \) be an edge which minimizes \( \text{dist}(s, \text{SP}(u, w)) \);
6: \( v := \text{arg min} \{\text{dist}(s, v) \mid v \in \text{SP}(u, w)\} \)
7: \( T := (V(T) \cup \{v\} \cup \{s\}, E(T) \setminus (u, w) \cup \{u, v\} \cup \{v, w\} \cup \{v, s\}) \)
8: \( S := S \setminus \{s\} \)
9: end while

In this notation \( \text{SP}(u, w) \subset \mathbb{R}^2 \) is the area covered by shortest paths between \( u \) and \( w \), and \( \text{dist}(s, T) \) is the minimum distance between \( s \) and the shortest path area \( \text{SP}(u, w) \) of an edge \( \{u, w\} \in E(T) \).

Show that the algorithm is a \( \frac{3}{2} \)-approximation algorithm for the MINIMUM STEINER TREE PROBLEM.

(Hint: First show that the length of \( T \) is at most the length of a minimum spanning tree on \( P \).) (6 points)
3. Implement a program that computes following items for a set of points in the plane $P$ w.r.t. $l_1$-distances.

(a) the bounding box netlength with a runtime of $O(n)$, (2 points)
(b) the clique netlength with a runtime of $O(n \log n)$, (2 points)
(c) the star netlength with a runtime of $O(n \log n)$, (2 points)
(d) the length of a minimum spanning tree with a runtime of $O(n^2)$. (4 points)
(e) a Steiner tree according to algorithm from Problem 2 with a runtime of $O(n^3)$. (6 points)

The implementation must be done either in the C++ or C programming language respecting the C/C++ standard from 1999. You can easily achieve this by using the GNU-compiler (gcc or g++) and by including only standard headers (including the STL).

The input should be read either from an input pipe or from a file, and write the 5 numbers to the standard output. Fill non-computed numbers with dummy tokens, if you are not able to implement all tasks. The data is given as a set of lines. Each line defines a pin through its the x/y-coordinates. The following instance defines an example with four pins.

```
0 0
1 1
0 1
1 0
```

The code must be sent to held@or.uni-bonn.de. On the exercise web page you will find a file with randomized instances and with VLSI instances.

```
http://www.or.uni-bonn.de/~held/vlsi_design_ss08/SteinerInstances.tar.gz
```

Run your program on these instances and create a table with the results. Furthermore run the GeoSteiner program on these instances and add the lengths of minimum Steiner trees to the table. You find GeoSteiner 3.1 on

```
http://www.diku.dk/geosteiner
```

The deadline for the problems 1. and 2. is **Tuesday May 6 at 12:15**, before the lecture.

The deadline for problem 3. (programming exercise) is **Tuesday May 13 at 12:15**, before the lecture.