Chip Design
SS 2008

Prof. Dr. Jens Vygen
Stephan Held

Exercise 3

1. For a finite set $V \subset \mathbb{R}^2$, the Voronoï diagram (w.r.t. to the l_2-norm) consists of the regions

$$P_v := \left\{ x \in \mathbb{R}^2 \;\middle|\; ||x - v||_2 = \min_{w \in V} ||x - w||_2 \right\}$$

for $v \in V$. The Delaunay triangulation of V is the graph

$$\Big(V, \Big\{ \{v, w\} \subset V, v \neq w, |P_v \cap P_w| > 1 \Big\} \Big).$$

   (a) Prove that every minimum spanning tree w.r.t. the l_2-norm is a subgraph of the Delaunay triangulation. (4 points)

   (b) Does the statement from 1a) hold when using the l_1-norm instead of the l_2-norm? (2 points)

2. Consider the following algorithm to compute a rectilinear Steiner tree T for a set P of points in the plane $\mathbb{R}^2$.

Choose $p \in P$ arbitrarily;
$T := (\{p\}, \emptyset), S := P \setminus \{p\}$
while $S \neq \emptyset$ **do**
    Choose $s \in S$ with minimum $dist(s, T)$;
    Let $\{u, w\} \in E(T)$ be an edge which minimizes $dist(s, SP(u, w))$;
    $v := \arg\min\{dist(s, v) \mid v \in SP(u, w)\}$
    $T := (V(T) \cup \{v\} \cup \{s\}, E(T) \setminus (u, w) \cup \{u, v\} \cup \{v, w\} \cup \{v, s\}\}$
    $S := S \setminus \{s\}$
end while

In this notation $SP(u, w) \subset \mathbb{R}^2$ is the area covered by shortest paths between u and w, and $dist(s, T)$ is the minimum distance between s and the shortest path area $SP(u, w)$ of an edge $\{u, w\} \in E(T)$.

Show that the algorithm is a $\frac{3}{2}$-approximation algorithm for the Minimum Steiner Tree Problem.

(Hint: First show that the length of T is at most the length of a minimum spanning tree on P.) (6 points)

3. Implement a program that computes following items for a set of points in the plane P w.r.t. l_1-distances.

   (a) the bounding box netlength with a runtime of $O(n)$, (2 points)

   (b) the clique netlength with a runtime of $O(n \log\ n)$, (2 points)

   (c) the star netlength with a runtime of $O(n \log\ n)$, (2 points)

   (d) the length of a minimum spanning tree with a runtime of $O(n^2)$. (4 points)

   (e) a Steiner tree according to algorithm from Problem 2 with a runtime of $O(n^3)$. (6 points)

   The implementation must be done either in the C++ or C programming language respecting the C/C++ standard from 1999. You can easily achieve this by using the GNU-compiler (gcc or g++) and by including only standard headers (including the STL).

   The input should be read either from an input pipe or from a file, and write the 5 numbers to the standard output. Fill non-computed numbers with dummy tokens, if you are not able to implement all tasks. The data is given as a set of lines. Each line defines a pin through its the x/y-coordinates. The following instance defines an example with four pins.

```
0 0
1 1
0 1
1 0
```

   The code must be sent to `held@or.uni-bonn.de`. On the exercise web page you will find a file with randomized instances and with VLSI instances.

   `http://www.or.uni-bonn.de/~held/vlsi_design_ss08/SteinerInstances.tar.gz`

   Run your program on these instances and create a table with the results. Furthermore run the GeoSteiner program on these instances and add the lengths of minimum Steiner trees to the table. You find GeoSteiner 3.1 on

   `http://www.diku.dk/geosteiner`

The deadline for the problems 1. and 2. is **Tuesday May 6 at 12:15**, before the lecture.
The deadline for problem 3. (programming exercise) is **Tuesday May 13 at 12:15**, before the lecture.