Exercise 2

1. Let $T$ be a Steiner tree on a terminal set $N$. Prove that

$$\sum_{v \in N} (|\delta_T(v)| - 1) = K_T - 1,$$

where $K_T$ is the number of full components in $T$. (6 points)

2. Prove that the following inequality holds for a net $N$ with at most five (point-shaped) pins:

$$\text{steiner}(N) \leq \frac{3}{2} \text{BB}(N).$$

(6 points)

3. Let $P$ be a set of pins, and let $\mathcal{R}_p$ be a set of axis-parallel rectangles for each $p \in P$. We search for a rectangle $B$ with minimum perimeter such that for every $p \in P$ there is an $R \in \mathcal{R}_p$ with $B \cap R \neq \emptyset$. We denote $n := \sum_{p \in P} |\mathcal{R}_p|$.  

(a) Show that such a rectangle $B$ can be computed in $O(n^4)$. (2 points)

(b) Show that such a rectangle $B$ can be computed in $O(n^3)$. 

(Hint: Enumerate all reasonable coordinates for the lower left corner of $B$.) (6 points)

The deadline of the exercise return is April 24, before the exercise.