Exercise 7.1: For \( n \in \mathbb{N} \), let \( P_n \) be the convex hull of all even 0-1-vectors. More precisely, let
\[
P_n = \text{conv}\{x \in \{0,1\}^n : \sum_{i=0}^n x_i \equiv 0 \pmod{2}\}.
\]
Prove \( rc(P_n) = 2^{\Theta(n)} \), i.e., \( P_n \) has an exponential relaxation complexity.

(4 Points)

Exercise 7.2: Let \( G \) be a connected graph, \( T \subseteq V(G) \) with \(|T|\) even, and \( F \subseteq E(G) \). A subset \( C \subseteq E(G) \) is called a \( T \)-cut if \( C = \delta(U) \) for some \( U \subseteq V(G) \) with \(|U \cap T|\) odd. Prove:

(i) \( F \) has nonempty intersection with every \( T \)-join if and only if \( F \) contains a \( T \)-cut.

(ii) \( F \) has nonempty intersection with every \( T \)-cut if and only if \( F \) contains a \( T \)-join.

(4 Points)

Exercise 7.3: Let \( G \) be a graph with edge weights \( c : E(G) \rightarrow \mathbb{R}_{>0} \). A set \( F \subseteq E(G) \) is called odd cover if the graph which results from \( G \) by successively contracting each \( e \in F \) is Eulerian. Show that it is possible in polynomial time to find an odd cover \( F \) that minimizes \( c(F) \) or to decide that none exists. We use the notation \( c(F) := \sum_{e \in F} c(e) \) for edge sets \( F \subseteq E(G) \).

(4 Points)

Exercise 7.4: Show that the following algorithm finds in a graph \( G \) (which is not a forest) with edge weights \( w : E(G) \rightarrow \mathbb{R} \) a cycle \( C \subseteq E(G) \) that minimizes \( \frac{w(C)}{|C|} \) in strongly polynomial time: First reduce all edge lengths by \( \max\{w(e) | e \in E(G)\} \). Then find a minimum-weight \( \emptyset \)-join \( J \). If \( w(J) = 0 \) output a cycle of length 0, otherwise add \( \frac{-w(J)}{|J|} \) to all edge lengths and iterate (i.e. find again a minimum-weight \( \emptyset \)-join).

(4 Points)

Deadline: Tuesday, November 25, 2014, before the lecture.

Information: Submissions by groups of one or two students are allowed.