Exercise 6.1: Let $G = (V, E)$ be an undirected graph and $Q$ its fractional perfect matching polytope, which is defined by

$$Q = \{ x \in \mathbb{R}^E : x_e \geq 0 \ (e \in E), \sum_{e \in \delta(v)} x_e = 1 \ (v \in V) \}. $$

If $G$ is bipartite, $Q$ is identical to the perfect matching polytope of $G$. Now, consider the first Gomory-Chvátal-truncation $Q'$ of $Q$. Prove that $Q'$ is always identical to the perfect matching polytope of $G$. \hspace{1cm} \text{(4 Points)}

Exercise 6.2: Let $G = (V, E)$ be an undirected graph and $n := |V|$. Prove that the spanning tree polytope of $G$ is in general a proper subset of the polytope

$$\{ x \in [0, 1]^E : \sum_{e \in E} x_e = n - 1, \sum_{e \in \delta(X)} x_e \geq 1 \text{ for } \emptyset \neq X \subset V \}. $$

\hspace{1cm} \text{(4 Points)}

Exercise 6.3: Let $G = (V, E)$ be an undirected graph and $n := |V|$. Prove that the following inequality system with $O(n^3)$ variables and constraints describes a polytope whose orthogonal projection to the $x$-variables yields the spanning tree polytope of $G$.

$$\begin{align*}
x_e & \geq 0 \quad (e \in E) \\
z_{u,v,w} & \geq 0 \quad (\{u, v\} \in E, w \in V \setminus \{u, v\}) \\
\sum_{e \in E} x_e & = n - 1 \\
x_e & = z_{u,v,w} + z_{v,u,w} \quad (e = \{u, v\} \in E, w \in V \setminus e) \\
x_e + \sum_{\{u,v\} \in \delta(v) \setminus \{e\}} z_{u,v,w} & = 1 \quad (v \in V, e = \{v, w\} \in \delta(v))
\end{align*} $$

\hspace{1cm} \text{(4 Points)}

Note the second page!
Exercise 6.4: Let $G = (V, E)$ be an undirected graph and $n := |V|$. Prove that the convex hull of the incidence vectors of all forests in $G$ is the polytope

$$\{ x \in [0, 1]^E : \sum_{e \in E(G[X])} x_e \leq |X| - 1 \text{ for } \emptyset \neq X \subseteq V \}. $$

(4 Points)

Deadline: Tuesday, November 18, 2014, before the lecture.

Information: Submissions by groups of one or two students are allowed.