Exercise 1.1: Find an infinite counterexample to Hall’s Theorem. More precisely: Find a bipartite graph $G = (A \cup B, E)$ with $A \cong \mathbb{N}$, $B \cong \mathbb{N}$, and $|\Gamma(S)| \geq |S|$ for every $S \subseteq A$ and every $S \subseteq B$ such that $G$ does not contain a perfect matching. (4 Points)

Exercise 1.2:
1. Let $M_1$ and $M_2$ be two maximal matchings in a graph $G$. Prove that $|M_1| \leq 2|M_2|$. (2 Points)

2. Let $G$ be a bipartite graph such that for each proper subset $F \subseteq E(G)$ and $G' := (V(G), F)$ we have $\tau(G') < \tau(G)$. Prove that $E(G)$ is a matching. (2 Points)

Exercise 1.3: Let $G$ be a bipartite graph. For each $v \in V(G)$, let $<_v$ be a linear ordering of $\delta(v)$. Prove that there is a matching $M \subseteq E(G)$ such that for each $e \in E(G) \setminus M$ there is an edge $f \in M$ and a vertex $v \in V(G)$ such that $v \in (e \cap f)$ and $e <_v f$. (4 Points)

Exercise 1.4:
Let $G$ be a graph. Prove following equalities:
1. $\alpha(G) + \tau(G) = |V(G)|$ for any graph $G$. (1 Points)

2. $\nu(G) + \zeta(G) = |V(G)|$ for any graph $G$ with no isolated vertices. (2 Points)

3. $\zeta(G) = \alpha(G)$ for any bipartite graph $G$ with no isolated vertices. (1 Points)

Deadline: Tuesday, October 14, 2014, before the lecture.
Information: submissions by groups of one or two students are allowed.