Exercise Set 11

Exercise 11.1. Let $x \in [0,1]^{E(K_n)}$ with $\sum_{e\in\delta(v)} x_e = 2$ for all $v \in V(K_n)$. Prove that if there exists a violated subtour constraint, i.e. a set $S \subset V(K_n)$ with $\sum_{e\in\delta(S)} x_e < 2$, then there exists one with $x_e < 1$ for all $e \in \delta(S)$.

(3 points)

Exercise 11.2. Let $n \in \mathbb{N}$. For which values of k is the uniform matroid of rank k on the set of n elements the graphic matroid of some simple graph?

(3 points)

Exercise 11.3. Let E be a finite set and $P \subseteq \mathbb{R}^E$ be a polymatroid. Show that there is some submodular set function f with $f(\emptyset) = 0$, f monotone, i.e. $f(X) \le f(Y)$ for all $X \subseteq Y \subseteq E$ and $P = P(f)$.

(4 points)

Exercise 11.4. Let E be a finite set and $f\colon 2^E \to \mathbb{R}$ a function and f' its Lovász extension. Prove Lemma 4.43 and Theorem 4.44 from the lecture, i.e. that

(i) if f is submodular, then for all $x \in [0,1]^E$

$$f'(x) = \max\{x^T y \colon y \in P(f)\};$$

(ii) f is submodular if and only if f' is convex.

(3+3 points)

Deadline: January 12, before the lecture. The websites for lecture and exercises can be found at:

`https://ecampus.uni-bonn.de/goto_ecampus_crs_2772883.html`

In case of any questions feel free to contact me at armbruster@or.uni-bonn.de.