Exercise Set 10

Exercise 10.1. For an undirected graph G, let P_G denote the spanning-tree polytope of G and

$$Q_G := \left\{ x \in [0,1]^{E(G)} : \sum_{e \in E(G)} x_e = |V(G)| - 1 \ , \sum_{e \in \delta(X)} x_e \geq 1 \text{ for } \emptyset \neq X \subsetneq V(G) \right\}.$$

Prove:

(i) $P_G \subseteq Q_G$ for every graph G.

(ii) There exists a graph G with $P_G \neq Q_G$.

(1+2 points)

Exercise 10.2. Let G be an undirected graph and and $n := |V(G)|$. Prove that the following linear inquality system with $\mathcal{O}(n^3)$ variables and constraints describes a polytope whose orthogonal projection onto the x-variables yields the spanning tree polyptope P_G of G.

$$\begin{aligned}
x_e &\geq 0 && (e \in E(G)) \\
z_{u,v,w} &\geq 0 && (\{u,v\} \in E(G), w \in V(G)) \\
z_{u,v,w} + z_{v,u,w} &= x_e && (e = \{u,v\} \in E(G), w \in V(G)) \\
\sum_{\{u,v\} \in \delta_G(v)} z_{u,v,w} &= 1 && (v \in V(G), w \in V(G) \setminus \{v\}) \\
\sum_{e \in E(G)} x_e &= n - 1
\end{aligned}$$

(5 points)

Exercise 10.3. Let $n \geq 4$ and $c : E(K_n) \to \mathbb{R}_{>0}$ be such that (K_n, c) is an instance of the METRIC TSP, and let T be a tour on K_n. Show that there is a tour $T' \neq T$ such that

$$|c(T') - c(T)| \leq \frac{2}{n} \cdot c(T).$$

(5 points)

Exercise 10.4. Santa Claus needs to distribute his presents to all houses in a city. The distances in the city are the metric closure of a weighted tree on all houses. Since the reindeers are already tired, it is important to find a shortest possible tour. Show that Santa can find the shortest tour in polynomial time and derive a (creative) corollary stating that his computation finishes before Christmas and he can deliver the presents on time.

(3 points)

Deadline: December 22, before the lecture. The websites for lecture and exercises can be found at:

`https://ecampus.uni-bonn.de/goto_ecampus_crs_2772883.html`

In case of any questions feel free to contact me at armbruster@or.uni-bonn.de.