Professor Dr. Stephan Held
Susanne Armbruster

Exercise Set 8

Exercise 8.1. Let G be an undirected graph, $T \subseteq V(G)$ and let $J \subseteq E(G)$ be a T-join of minimum cardinality. Denote by $\nu(G,T)$ the maximum cardinality of a family of pairwise disjoint T-cuts and by $\tau(G,T)$ the minimum cardinality of a T-join. Consider the properties:

(i) $\nu(G,T) = \tau(G,T)$.

(ii) There exist $|J|$ pairwise disjoint cuts $\delta(X_1), \dots, \delta(X_{|J|})$ with $|\delta(X_i) \cap J| = 1$ for every i.

Show that (i) and (ii) are equivalent.

(4 points)

Exercise 8.2. Find a (non-bipartite) graph G and a set $T \subseteq V(G)$ such that G contains a T-join and the minimum cardinality of a T-join in G is strictly larger than the maximum cardinality of a family of pairwise edge-disjoint T-cuts in G. You have to prove that the latter is true for the example you provide.

(3 points)

Exercise 8.3. Let $\lambda_{ij}, 1 \leq i, j \leq n$, be nonnegative numbers with $\lambda_{ij} = \lambda_{ji}$ and $\lambda_{ik} \geq \min\{\lambda_{ij}, \lambda_{jk}\}$ for any three distinct indices $i, j, k \in \{1, \dots, n\}$. Show that there exists a graph G with $V(G) = \{1, \dots, n\}$ and capacities $u \colon E(G) \to \mathbb{R}_+$ such that the local edge-connectivities are precisely the λ_{ij}.

Hint: Consider a maximum weight spanning tree in (K_n, c), where $c(\{i,j\}) := \lambda_{ij}$.

(4 points)

Exercise 8.4. Given an undirected graph G and disjoint sets $S_e, S_o \subseteq V(G)$, a *partial* (S_e, S_o)*-join* is a set $J \subseteq E(G)$ such that $|\delta(v) \cap J|$ is even for every $v \in S_e$ and odd for every $v \in S_o$. (In particular, a T-join is the same as a partial $(V(G) \setminus T, T)$-join.) Consider the Minimum Weight Partial (S_e, S_o)-Join Problem: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and disjoint sets $S_e, S_o \subseteq V(G)$, find a partial (S_e, S_o)-join of minimum weight, or determine that none exists. Show that this problem can be linearly reduced to the Minimum Weight T-Join Problem.

(5 points)

Professor Dr. Stephan Held
Susanne Armbruster

Deadline: December 8, before the lecture. The websites for lecture and exercises can be found at:

`https://ecampus.uni-bonn.de/goto_ecampus_crs_2772883.html`

In case of any questions feel free to contact me at armbruster@or.uni-bonn.de.