Exercise Set 7

Exercise 7.1. Let $k \in \mathbb{N}$, $k \geq 1$, and suppose G is a k-regular and $(k-1)$-edge-connected graph with an even number of vertices, and with edge weights $c : E(G) \to \mathbb{R}$. Show that there is a perfect matching M in G with
$c(M) \leq (1/k) \cdot c(E(G))$.

(4 points)

Exercise 7.2. Let G be a graph and $T \subseteq V(G)$ with $|T|$ even. Prove:

(i) A set $F \subseteq E(G)$ intersects every T-join if and only if it contains a T-cut.

(ii) A set $F \subseteq E(G)$ intersects every T-cut if and only if it contains a T-join.

(4 points)

Exercise 7.3. Consider the Directed Chinese Postman Problem: Given a strongly connected simple digraph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find a function $f : E(G) \to \mathbb{N} \setminus \{0\}$ such that if each edge $e \in E(G)$ is replaced by $f(e)$ copies of itself, the resulting graph is Eulerian, and such that f minimizes $\sum_{e \in E(G)} f(e)c(e)$ among functions with this property. Show that this problem can be linearly reduced to the Minimum Cost Integral Flow Problem (i.e. the Minimum Cost Flow Problem with the additional requirement that the flow must be integral).

(4 points)

Exercise 7.4. The Undirected Minimum Mean-Weight Cycle Problem is the following: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}$, find a cycle C whose mean-weight $c(E(C))/|E(C)|$ is minimum, or determine that G is acyclic. Consider the following algorithm for the Undirected Minimum Mean-Weight Cycle Problem: First determine with a linear search whether G has cycles or not, and if not return with this information.
Let $\gamma := \max\{c(e) : e \in E(G)\}$ and define a new edge-weight function via $c'(e) := c(e) - \gamma$. Let $T := \emptyset$. Now iterate the following: Find a minimum c'-weight T-join J with a polynomial (black-box) algorithm. If $c'(J) = 0$, return any zero-c'-weight cycle. Otherwise, let $\gamma' := c'(J)/|J|$, reset c' via $c'(e) \leftarrow c'(e) - \gamma'$, and continue.

Show that this algorithm works correctly and runs in polynomial time. Also, explain how to get the cycle to be returned in the case $c'(J) = 0$.

(4 points)

Deadline: December 1, before the lecture. The websites for lecture and exercises can be found at:

`https://ecampus.uni-bonn.de/goto_ecampus_crs_2772883.html`

In case of any questions feel free to contact me at armbruster@or.uni-bonn.de.