Professor Dr. Stephan Held
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Exercise Set 6

Exercise 6.1. Consider the SHORTEST EVEN/ODD PATH PROBLEM: Given a graph G with weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and $s, t \in V(G)$, find an s-t-path P of even/odd length in G that minimizes $\sum_{e \in E(P)} c(e)$ among all s-t-paths of even/odd length in G. Show that both the even and the odd version can be linearly reduced to the MINIMUM WEIGHT PERFECT MATCHING PROBLEM.

(4 points)

Exercise 6.2. Let G be a graph with edge weights $c : E(G) \to \mathbb{R}$ and let M be a matching in G with $|M| = k$ that has minimum weight among all matchings in G that contain exactly k edges. Let P be an M-augmenting path in G with minimum gain. Let $M' := M \triangle E(P)$. Prove that M' has minimum weight among all matchings in G that contain exactly $k + 1$ edges.

(4 points)

Exercise 6.3. Let $G = (V, E)$ be an undirected graph and Q its fractional perfect matching polytope, which is defined by

$$Q = \{x \in \mathbb{R}^E : x_e \geq 0 \ (e \in E), \sum_{e \in \delta(v)} x_e = 1 \ (v \in V)\}.$$

Prove that a vector $x \in Q$ is a vertex of Q if and only if there exist vertex disjoint odd circuits $C_1, \ldots, C_k$ and a perfect matching M in $G - (V(C_1) \cup \ldots \cup V(C_k))$ such that

$$x_e = \begin{cases} \frac{1}{2} & \text{if } e \in E(C_1) \cup \ldots \cup E(C_k), \\ 1 & \text{if } e \in M, \\ 0 & \text{otherwise.} \end{cases}$$

(4 points)

Exercise 6.4. Let $n \in \mathbb{N}$. A graph with $2n + 1$ vertices is called a *double star* if it emerges from a star with $n + 1$ vertices by replacing every edge $\{v, w\}$ by a vertex z_{vw} and two edges $\{v, z_{vw}\}, \{z_{vw}, w\}$.
Show that there exists a polynomial time algorithm that, given a cost function c

on the edges of the complete graph K_{2n+1}, finds a spanning double star S of K_{2n+1} that minimizes $c(E(S))$.

(4 points)

Deadline: November 24, before the lecture. The websites for lecture and exercises can be found at:

`https://ecampus.uni-bonn.de/goto_ecampus_crs_2772883.html`

In case of any questions feel free to contact me at armbruster@or.uni-bonn.de.