Professor Dr. Stephan Held
Susanne Armbruster

Exercise Set 5

Exercise 5.1. Let G be a 2-edge-connected graph, and let $\varphi(G)$ be the minimum number of even ears in any ear-decomposition of G. Show that then for every $v \in V(G)$ there is a matching in $G - v$ of cardinality $\frac{1}{2}\Big(n - 1 - \varphi(G)\Big)$.

(4 points)

Exercise 5.2. Let G be a simple, bridgeless, 3-regular graph.

(a) Prove that G has a perfect matching.

(b) Is there a simple, 3-regular graph without a perfect matching?

(3+1 points)

Exercise 5.3. Show that for every $k \in \mathbb{N}$ there exists a graph G of minimum degree k such that G has exactly one perfect matching.

(3 points)

Exercise 5.4. Consider the Minimum Cost Edge Cover Problem: Given a graph G with weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. Show that the Minimum Cost Edge Cover Problem can be linearly reduced to the Minimum Weight Perfect Matching Problem.

(5 points)

Deadline: November 17, before the lecture. The websites for lecture and exercises can be found at:

`https://ecampus.uni-bonn.de/goto_ecampus_crs_2772883.html`

In case of any questions feel free to contact me at armbruster@or.uni-bonn.de.