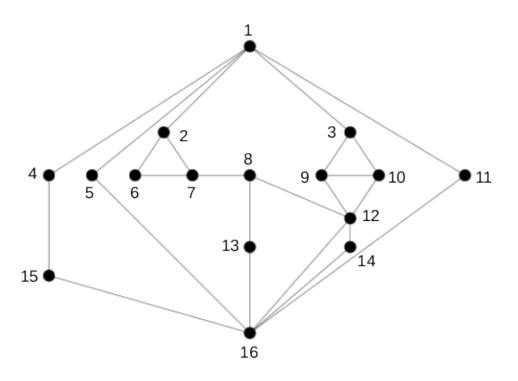
## Exercise Set 4

Figure 4.1: Graph for exercise 4.1



**Exercise 4.1.** Find a maximum matching in the graph in Figure 4.1 together with a certificate for its maximality.

(3 points)

**Exercise 4.2.** Show that a graph G is factor-critical if and only if G is connected and for every vertex  $v \in V(G)$  we have  $\nu(G - v) = \nu(G)$ .

(4 points)

**Exercise 4.3.** Let G be a 3-regular undirected graph.

- (a) Assume G is simple. Show that there is a matching in G covering at least  $(7/8) \cdot |V(G)|$  vertices.
- (b) Give an example to prove that the bound of item (a) is tight.

(c) Show that the assumption that G is simple in item (a) is necessary.

(3+1+1 points)

**Exercise 4.4.** Let G be a k-vertex-connected graph which has neither a perfect nor a near-perfect matching.

- (i) Show that  $\nu(G) \ge k$ .
- (ii) Show that  $\tau(G) \leq 2 \cdot \nu(G) k$ .

(2+2 points)

**Deadline:** November 10, before the lecture. The websites for lecture and exercises can be found at:

https://ecampus.uni-bonn.de/goto\_ecampus\_crs\_2772883.html

In case of any questions feel free to contact me at armbruster@or.uni-bonn.de.