Professor Dr. Stephan Held
Susanne Armbruster, M. Sc.

Exercise Set 1

Exercise 1.1. Let $\alpha(G)$ denote the size of a maximum stable set in G, and $\zeta(G)$ the minimum cardinality of an edge cover. Prove:

(a) $\alpha(G) + \tau(G) = |V(G)|$ for any graph G,

(b) $\nu(G) + \zeta(G) = |V(G)|$ for any graph G with no isolated vertices,

(c) $\zeta(G) = \alpha(G)$ for any bipartite graph G with no isolated vertices.

(1+2+1 points)

Exercise 1.2. Let G be a graph and M_1 and M_2 be two inclusion-wise maximal matchings in G. Prove that $|M_1| \leq 2|M_2|$.

(3 points)

Exercise 1.3. Let G be a k-regular bipartite graph.

(a) Prove that G contains k disjoint perfect matchings.
Hint: Use König's Theorem.

(b) Deduce from (a) that the edge set of any bipartite graph of maximum degree k can be partitioned into k matchings.

(2+3 points)

Exercise 1.4. Find an infinite counterexample to Hall's Theorem. More precisely: Find a bipartite graph $G = (A \dot{\cup} B, E)$ with $A \cong \mathbb{N}$, $B \cong \mathbb{N}$, and $|N(S)| \geq |S|$ for every $S \subseteq A$ and every $S \subseteq B$ such that G does not contain a perfect matching.

(4 points)

Deadline: October 20, 2:15 pm. Further information on lecture and exercises can be found in the corresponding eCampus course at:

`https://ecampus.uni-bonn.de/goto_ecampus_crs_2772883.html`

Since there will be no lecture on October 20, please hand in the solution via email to armbruster@or.uni-bonn.de. Feel free to also contact me in case of questions.