Professor Dr. Stephan Held
Stefan Rabenstein

Exercise Set 9

Exercise 9.1. Let $n \geq 3$ and $x : E(K_n) \to [0,1]$ be such that it satisfies all degree constraints of the TSP but not all subtour elimination constraints. Show that there is a non-empty set $S \subsetneq V(K_n)$ such that

$$\sum_{e \in E(K_n[S])} x_e > |S| - 1$$

and $x_e < 1$ for all $e \in \delta(S)$.

(4 points)

Exercise 9.2. Consider the 2-matching inequalities

$$\sum_{e \in E(K_n[X]) \cup F} x_e \leq |X| + \frac{|F|-1}{2} \qquad \text{for } X \subseteq V(K_n), F \subseteq \delta(X) \text{ with } |F| \text{ odd.}$$

Show that if a vector in the subtour polytope satisfies all 2-matching inequalities where F is a matching, then it satisfies all 2-matching inequalities.

(6 points)

Exercise 9.3. Given an instance (K_n, c) of the TSP, denote by $HK(K_n, c)$ the Held-Karp lower bound and by $\mathrm{opt}(K_n, c)$ the length of an optimum tour. Show that for instances of the Metric TSP the ratio $\mathrm{opt}(K_n, c)/HK(K_n, c)$ can be arbitrarily close to $4/3$.

(6 points)

Exercise 9.4. Consider the Nearest Neighbor Heuristic (NNH): Given an instance (K_n, c) of the TSP, choose some $v_1 \in V(K_n)$. For $i = 2, \ldots, n$, choose $v_i \in V(K_n) \setminus \{v_1, \ldots, v_{i-1}\}$ such that $c(\{v_{i-1}, v_i\})$ is smallest possible. Return the tour given by the vertex sequence $(v_1, \ldots, v_n)$. Denote by $\mathrm{opt}^{NNH}(K_n, c)$ the shortest possible length of any tour returned by the Nearest Neighbor Heuristic (i.e., taking the minimum over all possible choices within the algorithm), and by $\mathrm{opt}(K_n, c)$ the length of an optimum tour. Show that the ratio $\mathrm{opt}^{NNH}(K_n, c)/\mathrm{opt}(K_n, c)$ can be arbitrarily large.

(4 points)

Professor Dr. Stephan Held
Stefan Rabenstein

Deadline: December 12$^{\text{th}}$, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws19/co_exercises/exercises.html`

In case of any questions feel free to contact me at rabenstein@or.uni-bonn.de.