Exercise 4.1. Consider the Shortest Even/Odd Path Problem: Given a graph $G$ with weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and $s, t \in V(G)$, find an $s$-$t$-path $P$ of even/odd length in $G$ that minimizes $\sum_{e \in E(P)} c(e)$ among all $s$-$t$-paths of even/odd length in $G$. Show that both the even and the odd version can be linearly reduced to the Minimum Weight Perfect Matching Problem. (5 points)

Exercise 4.2. Consider the Minimum Cost Edge Cover Problem: Given a graph $G$ with weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. Show that the Minimum Cost Edge Cover Problem can be linearly reduced to the Minimum Weight Perfect Matching Problem. (5 points)

Exercise 4.3. Let $G$ be a graph with edge weights $c : E(G) \to \mathbb{R}$ and let $M$ be a matching in $G$ with $|M| = k$ that has minimum weight among all matchings in $G$ that contain exactly $k$ edges. Let $P$ be an $M$-augmenting path in $G$ with minimum gain. Let $M' := M \triangle E(P)$. Prove that $M'$ has minimum weight among all matchings in $G$ that contain exactly $k + 1$ edges. (5 points)

Exercise 4.4. Let $G$ be an undirected graph with edge weights $c : E(G) \to \mathbb{R}$, and let $M$ be a matching so that $c(N) \leq c(M)$ for all matchings $N$ in $G$ with $|M| - 1 \leq |N| \leq |M| + 1$. Prove that then $M$ is a maximum weight matching in $G$. (5 points)

Deadline: November 7th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ws19/co_exercises/exercises.html

In case of any questions feel free to contact me at rabenstein@or.uni-bonn.de.