Professor Dr. Jens Vygen
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Exercise Set 10

Exercise 10.1. Consider the 2-matching inequalities

$$\sum_{e \in E(K_n[X]) \cup F} x_e \leq |X| + \frac{|F|-1}{2} \qquad \text{for } X \subseteq V(K_n), F \subseteq \delta(X) \text{ with } |F| \text{ odd.}$$

Show that if a vector in the subtour polytope satisfies all 2-matching inequalities where F is a matching, then it satisfies all 2-matching inequalities.

(5 points)

Exercise 10.2. Let $x \in [0,1]^{E(K_n)}$ with $\sum_{e \in \delta(v)} x_e = 2$ for all $v \in V(K_n)$. Prove that if there exists a violated subtour constraint, i.e. a set $S \subset V(K_n)$ with $\sum_{e \in \delta(S)} x_e < 2$, then there exists one with $x_e < 1$ for all $e \in \delta(S)$.

(5 points)

Exercise 10.3. Let $(E, \mathcal{F})$ be a clutter. Show that the blocking clutter of the blocking clutter of $(E, \mathcal{F})$ equals $(E, \mathcal{F})$.

(5 points)

Exercise 10.4. Let U be a finite set and $f\colon 2^U \to \mathbb{R}$. Prove that f is submodular if and only if $f(X \cup \{y,z\}) - f(X \cup \{y\}) \leq f(X \cup \{z\}) - f(X)$ for all $X \subseteq U$ and $y, z \in U$ with $y \neq z$.

(5 points)

Information: Submissions in groups of up to two students are allowed.

Deadline: Tuesday, January 8, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.