Professor Dr. Jens Vygen
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Exercise Set 9

Exercise 9.1. Let $\lambda_{ij}, 1 \leq i, j \leq n$, be nonnegative numbers with $\lambda_{ij} = \lambda_{ji}$ and $\lambda_{ik} \geq \min\{\lambda_{ij}, \lambda_{jk}\}$ for any three distinct indices $i, j, k \in \{1, \ldots, n\}$. Show that there exists a graph G with $V(G) = \{1, \ldots, n\}$ and capacities $u\colon E(G) \to \mathbb{R}_+$ such that the local edge-connectivities are precisely the λ_{ij}.

Hint: Consider a maximum weight spanning tree in (K_n, c), where $c(\{i, j\}) := \lambda_{ij}$.

(5 points)

Exercise 9.2. Let G be an undirected graph and $T \subseteq V(G)$ with $|T| = 2k$ even. Prove that the minimum cardinality of a T-cut in G equals the maximum of $\min_{i=1}^{k} \lambda_{s_i,t_i}$ over all pairings $T = \{s_1, t_1, \ldots, s_k, t_k\}$, where $\lambda_{s,t}$ denotes the maximum number of pairwise edge-disjoint s-t-paths.

(5 points)

Information: Submissions in groups of up to two students are allowed.

Deadline: Tuesday, December 18, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.