Exercise Set 8

Exercise 8.1. Describe a set of instances of the Metric TSP for which Christofides’ Algorithm returns a tour whose length is arbitrarily close to \( \frac{3}{2} \) times the optimum. 

(4 points)

Exercise 8.2. For an undirected graph \( G \), let \( P_G \) denote the spanning-tree polytope of \( G \) and

\[
Q_G := \left\{ x \in [0, 1]^{|E(G)|} : \sum_{e \in E(G)} x_e = |V(G)| - 1 , \sum_{e \in \delta(X)} x_e \geq 1 \text{ for } \emptyset \neq X \subsetneq V(G) \right\}.
\]

Prove:

(i) \( P_G \subseteq Q_G \) for every graph \( G \).

(ii) There exists a graph \( G \) with \( P_G \neq Q_G \).

(2 + 2 points)

Exercise 8.3. Let \( G \) be a 2-edge-connected graph, and let \( T := \{ v \in V(G) : |\delta(v)| \text{ odd} \} \).

(i) Prove that \( x \in \mathbb{R}^{E(G)} \) with \( x_e = \frac{1}{3} \) for all \( e \in E(G) \) is a convex combination of incidence vectors of \( T \)-joins.

(ii) Show that a connected Eulerian subgraph \( H \) of \( 2G \) with \( V(H) = V(G) \) and \( |E(H)| \leq \frac{2}{3} \left( |V(G)| + |E(G)| - 1 \right) \) can be computed in polynomial time.

(4 + 3 points)

Information: Submissions in groups of up to two students are allowed.
Deadline: Tuesday, December 11, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ws18/coex.html

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.