Professor Dr. Jens Vygen
Rudolf Scheifele

Exercise Set 8

Exercise 8.1. Describe a set of instances of the METRIC TSP for which Christofides' Algorithm returns a tour whose length is arbitrarily close to $\frac{3}{2}$ times the optimum.

(4 points)

Exercise 8.2. For an undirected graph G, let P_G denote the spanning-tree polytope of G and

$$Q_G := \left\{ x \in [0,1]^{E(G)} : \sum_{e \in E(G)} x_e = |V(G)| - 1 \, , \sum_{e \in \delta(X)} x_e \geq 1 \text{ for } \emptyset \neq X \subsetneq V(G) \right\}.$$

Prove:

(i) $P_G \subseteq Q_G$ for every graph G.

(ii) There exists a graph G with $P_G \neq Q_G$.

(2 + 2 points)

Exercise 8.3. Let G be a 2-edge-connected graph, and let $T := \{v \in V(G) : |\delta(v)| \text{ odd}\}$.

(i) Prove that $x \in \mathbb{R}^{E(G)}$ with $x_e = \frac{1}{3}$ for all $e \in E(G)$ is a convex combination of incidence vectors of T-joins.

(ii) Show that a connected Eulerian subgraph H of $2G$ with $V(H) = V(G)$ and $|E(H)| \leq \frac{2}{3}\Big(|V(G)| + |E(G)| - 1\Big)$ can be computed in polynomial time.

(4 + 3 points)

Information: Submissions in groups of up to two students are allowed.

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Deadline: Tuesday, December 11, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.