Professor Dr. Jens Vygen
Rudolf Scheifele

Exercise Set 7

Exercise 7.1. Given an undirected graph G and disjoint sets $S_e, S_o \subseteq V(G)$, a *partial* (S_e, S_o)*-join* is a set $J \subseteq E(G)$ such that $|\delta(v) \cap J|$ is even for every $v \in S_e$ and odd for every $v \in S_o$. (In particular, a T-join is the same as a partial $(V(G) \setminus T, T)$-join.) Consider the MINIMUM WEIGHT PARTIAL (S_e, S_o)-JOIN PROBLEM: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and disjoint sets $S_e, S_o \subseteq V(G)$, find a partial (S_e, S_o)-join of minimum weight, or determine that none exists. Show that this problem can be linearly reduced to the MINIMUM WEIGHT T-JOIN PROBLEM.

(5 points)

Exercise 7.2. Let G be a graph and $T \subseteq V(G)$ with $|T|$ even. Prove:

(i) A set $F \subseteq E(G)$ intersects every T-join if and only if it contains a T-cut.

(ii) A set $F \subseteq E(G)$ intersects every T-cut if and only if it contains a T-join.

(2 + 2 points)

Exercise 7.3. The UNDIRECTED MINIMUM MEAN-WEIGHT CYCLE PROBLEM is the following: Given an undirected graph G with edge-weights $c : E(G) \to \mathbb{R}$, find a cycle C whose mean-weight $c(E(C))/|E(C)|$ is minimum, or determine that G is acyclic. Consider the following algorithm for the UNDIRECTED MINIMUM MEAN-WEIGHT CYCLE PROBLEM: First determine with a linear search whether G has cycles or not, and if not return with this information. Let $\gamma := \max\{c(e) : e \in E(G)\}$ and define a new edge-weight function via $c'(e) := c(e) - \gamma$. Let $T := \emptyset$. Now iterate the following: Find a minimum c'-weight T-join J with a polynomial (black-box) algorithm. If $c'(J) = 0$, return any zero-c'-weight cycle. Otherwise, let $\gamma' := c'(J)/|J|$, reset c' via $c'(e) \leftarrow c'(e) - \gamma'$, and continue.

Show that this algorithm works correctly and runs in polynomial time. Also, explain how to the get the cycle to be returned in the case $c'(J) = 0$.

(6 points)

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Exercise 7.4. Consider the metric s-t path TSP: Given an instance of METRIC TSP and two vertices s and t, we look for a Hamiltonian s-t path of minimum weight. Describe a $\frac{5}{3}$-factor approximation algorithm, generalizing Christofides' Algorithm.

(5 points)

Information: Submissions in groups of up to two students are allowed.

Deadline: Tuesday, December 4, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.