Professor Dr. Jens Vygen
Rudolf Scheifele

Exercise Set 6

Exercise 6.1. Let $k \in \mathbb{N}$, $k \geq 1$, and let G be a k-regular and $(k-1)$-edge-connected graph with an even number of vertices.

(a) Show that $\frac{1}{k}\mathbb{1}$ is in the perfect matching polytope, where $\mathbb{1}$ is the vector whose entries are all 1.

(b) Deduce from (a) that for any weights $c\colon E(G) \to \mathbb{R}$ there is a perfect matching M in G with $c(M) \leq \frac{1}{k}c(E(G))$.

(4 + 1 points)

Exercise 6.2. Consider the Shortest Even/Odd Path Problem: Given a graph G with weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and $s, t \in V(G)$, find an s-t-path P of even/odd length in G that minimizes $\sum_{e \in E(P)} c(e)$ among all s-t-paths of even/odd length in G. Show that both the even and the odd version can be polynomially reduced to the Minimum Weight Perfect Matching Problem.

(5 points)

Exercise 6.3. Let G be an undirected graph and $b_1, b_2\colon V(G) \to \mathbb{Z}_{\geq 0}$. Describe the convex hull of functions $f\colon E(G) \to \mathbb{Z}_{\geq 0}$ with $b_1(v) \leq \sum_{e \in \delta(v)} f(e) \leq b_2(v)$.

Hint: For $X, Y \subseteq V(G)$ with $X \cap Y = \emptyset$ consider the constraint

$$\sum_{e \in E(G[X])} f(e) - \sum_{e \in E(G[Y]) \cup E(Y,Z)} f(e) \leq \left\lfloor \frac{1}{2}\left(\sum_{x \in X} b_2(x) - \sum_{y \in Y} b_1(y)\right)\right\rfloor,$$

where $Z := V(G) \setminus (X \cup Y)$.

(5 points)

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Exercise 6.4. Consider the DIRECTED CHINESE POSTMAN PROBLEM: Given a strongly connected simple digraph G with edge-weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find a function $f : E(G) \to \mathbb{N} \setminus \{0\}$ such that if each edge $e \in E(G)$ is replaced by $f(e)$ copies of itself, the resulting graph is Eulerian, and such that f minimizes $\sum_{e \in E(G)} f(e)c(e)$ among functions with this property. Show that this problem can be linearly reduced to the MINIMUM COST INTEGRAL FLOW PROBLEM (i.e. the MINIMUM COST FLOW PROBLEM with the additional requirement that the flow must be integral).

(5 points)

Information: Submissions in groups of up to two students are allowed.

Deadline: Tuesday, November 27, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.