Professor Dr. Jens Vygen
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Exercise Set 5

Exercise 5.1. Show that $|\Omega| \leq \frac{3}{2}|V(G)|$ holds throughout Edmonds' Minimum Weight Perfect Matching Algorithm.

(5 points)

Exercise 5.2. Consider the Minimum Cost Edge Cover Problem: Given a graph G with weights $c\colon E(G) \to \mathbb{R}_{\geq 0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. Show that the Minimum Cost Edge Cover Problem can be solved in polynomial time.

(5 points)

Exercise 5.3. Let $G = (V, E)$ be an undirected graph and P be the polytope defined by

$$P = \{x \in \mathbb{R}^E : x_e \geq 0 \ (e \in E), \sum_{e \in \delta(v)} x_e = 1 \ (v \in V)\}.$$

Prove that a vector $x \in P$ is a vertex of P if and only if there exist vertex-disjoint odd circuits $C_1, \ldots, C_k$ and a perfect matching M in $G - (V(C_1) \cup \ldots \cup V(C_k))$ such that

$$x_e = \begin{cases} \frac{1}{2} & \text{if } e \in E(C_1) \cup \ldots \cup E(C_k), \\ 1 & \text{if } e \in M, \\ 0 & \text{otherwise.} \end{cases}$$

(5 points)

Information: Submissions in groups of up to two students are allowed.

Deadline: Tuesday, November 20, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.