Professor Dr. Jens Vygen
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Exercise Set 4

Exercise 4.1. Let G be a k-vertex-connected graph which has neither a perfect nor a near-perfect matching.

(i) Show that $\nu(G) \geq k$.

(ii) Show that $\tau(G) \leq 2 \cdot \nu(G) - k$.

(2+2 points)

Exercise 4.2. Let G be an undirected graph with edge weights $c\colon E(G) \to \mathbb{R}$, and let M be a matching so that $c(N) \leq c(M)$ for all matchings N in G with $|M| - 1 \leq |N| \leq |M| + 1$. Prove that then M is a maximum weight matching in G.

(5 points)

Exercise 4.3. Let G be an undirected graph and $c_1, c_2\colon E(G) \to \mathbb{R}$ be two weight functions. Let $\mathcal{M}$ be the set of all matchings that have maximum weight with respect to c_1. How can we find, in polynomial time, a matching $M \in \mathcal{M}$ such that $c_2(M)$ is maximum among all matchings in $\mathcal{M}$? Can you devise a strongly polynomial algorithm? (For this, in particular, the algorithm should work for arbitrary real numbers, assuming that we can perform addition, subtraction and comparison.)

Note: You can use the fact that there exists a strongly polynomial algorithm for the Maximum Weight Matching Problem.

(6 points)

Information: Submissions in groups of up to two students are allowed.

Deadline: Tuesday, November 13, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.