Professor Dr. Jens Vygen
Rudolf Scheifele

Exercise Set 1

Exercise 1.1. Let G be a graph and M_1 and M_2 be two inclusion-wise maximal matchings in G. Prove that $|M_1| \leq 2|M_2|$.

(4 points)

Exercise 1.2. Let $\alpha(G)$ denote the size of a maximum stable set in G, and $\zeta(G)$ the minimum cardinality of an edge cover. Prove:

(a) $\alpha(G) + \tau(G) = |V(G)|$ for any graph G,

(b) $\nu(G) + \zeta(G) = |V(G)|$ for any graph G with no isolated vertices,

(c) $\zeta(G) = \alpha(G)$ for any bipartite graph G with no isolated vertices.

(1 + 2 + 1 points)

Exercise 1.3. Let G be a k-regular bipartite graph.

(a) Prove that G contains k disjoint perfect matchings.
Hint: Use König's Theorem.

(b) Deduce from (a) that the edge set of any bipartite graph of maximum degree k can be partitioned into k matchings.

(2 + 3 points)

Exercise 1.4. Given a directed graph G, edge capacities $u\colon E(G) \to \mathbb{Z}_{\geq 0}$ and $s, t \in V(G)$, consider the linear programming formulation of the Maximum Flow Problem:

$$\begin{array}{lll}
\max & \displaystyle\sum_{e \in \delta^+(s)} x_e - \sum_{e \in \delta^-(s)} x_e & \\
\text{s.t.} & \displaystyle\sum_{e \in \delta^+(v)} x_e = \sum_{e \in \delta^-(v)} x_e & \text{for all } v \in V(G) \setminus \{s, t\} \\
& x_e \leq u(e) & \text{for all } e \in E(G) \\
& x_e \geq 0 & \text{for all } e \in E(G)
\end{array}$$

Show that the dual LP always has an integral optimum solution, and deduce the Max-Flow Min-Cut Theorem from this.

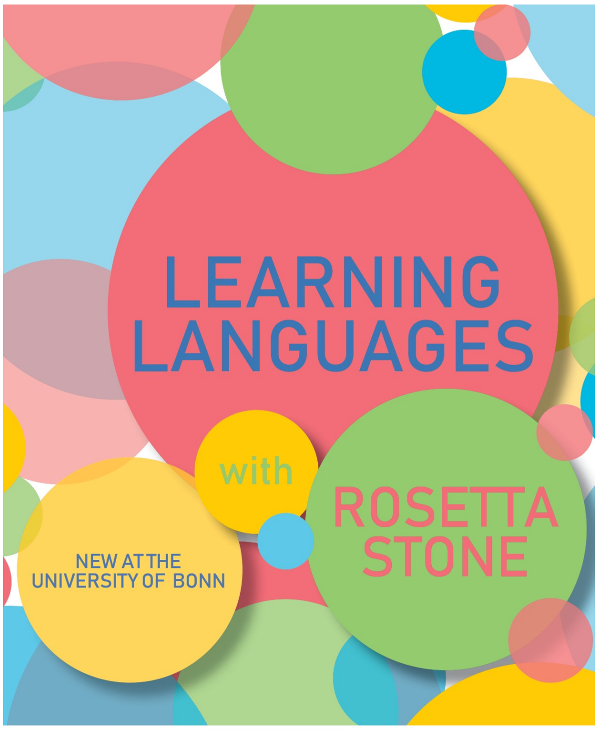
Hint: Use the complementary slackness conditions.

(7 points)

Deadline: October 18, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws18/coex.html`

In case of any questions feel free to contact me at scheifele@or.uni-bonn.de.



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