Winter semester 2017/18
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Linear and Integer Optimization

Exercise Sheet 13

Exercise 13.1: Let P the convex hull of the three points $(0,0)$, $(1,0)$, and $(\frac{1}{2}, k)$ in $\mathbb{R}^2$, where $k \in \mathbb{N}$. Prove that $P^{(2k-1)} \neq P_I$, but $P^{(2k)} = P_I$. (4 Points)

Exercise 13.2: Let $P \subseteq [0,1]^n$ be a polytope in the unit hypercube with $P_I = \emptyset$. Prove that $P^{(n)} = \emptyset$. (4 Points)

Exercise 13.3 Let $S := \{x \in \mathbb{Z}^2_+ \ : \ 4x_1 + x_2 \leq 28, x_1 + 4x_2 \leq 27, x_1 - x_2 \leq 1\}$. Describe the facets of conv(S) by (iteratively) applying Gomory-Chvátal cuts. (A few cuts are sufficient, you may first find the facets geometrically) (4 Points)

Exercise 13.4 Let $N := \{1, \dots, n\}$ for $n \in \mathbb{N}$ and $c : N^2 \mapsto \mathbb{R}$. Consider the following ordering problem for computing a maximum-cost permutation $\pi : N \to N$:

$$\begin{aligned} \max \sum_{i,j \in N} c_{ij} x_{ij} & & \\ x_{ij} + x_{ji} &= 1 & \forall\ 1 \leq i < j \leq n \\ x_{i_1 i_2} + \cdots + x_{i_r i_1} &\leq |C| - 1 & \forall C = \{j_1, \dots, j_r\} \\ x &\in \{0,1\}^{n \times n}. & \end{aligned}$$

Here $x_{ij} = 1$ for $i \neq j$ can be interpreted as $\pi(i) < \pi(j)$. The set of inequalities are called cycle inequalities.

1. Why does x determine a permutation?
2. How can the IP be used to compute a maximum-cost acyclic orientation of a complete graph with edge cost?
3. Prove that the cycle inequalities with $|C| \geq 4$ are redundant.
4. Prove that the cycle inequalities with $|C| = 3$ are facet-definint.

(4 Points)

Submission deadline: Thursday, January 25, 2018, before the lecture (in groups of 2 students).