Winter semester 2017/18
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Linear and Integer Optimization

Exercise Sheet 12

Exercise 12.1: Let $A \in \mathbb{Z}^{m \times n}$ and $b \in \mathbb{Z}^m$. Prove that there is a polynomial-time algorithm that either returns an integral solution x fo $Ax = b$ or decides that no integral solution exists. (4 Points)

Exercise 12.2: Prove that the system

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

is not TDI. Furthermore, specify a TDI-system representing the same polyhedron (including a proof that it is TDI). (4 Points)

Exercise 12.3: Let $a \neq 0$ be an integral vector and β a rational number. Prove that the inequality $a^\intercal x \leq \beta$ is TDI if and only if the components of a are relatively prime, i.e. their greatest common divisor is 1. (4 Points)

Exercise 12.4:
Show that $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is not totally unimodular, but $\{x : Ax = b\}$ is integral for all integral vectors b. (4 Points)

Submission deadline: Thursday, January 18, 2018, before the lecture (in groups of 2 students).