Winter semester 2017/18
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Linear and Integer Optimization

Exercise Sheet 9

Exercise 9.1: Use the Ellipsoid Algorithm to show that a given feasible and bounded linear program $\max\{c^t x \mid Ax \leq b\}$ with $A \in \mathbb{Q}^{m\times n}$, $b \in \mathbb{Q}^m$ and $c \in \mathbb{Q}^n$ can be solved in time $O((m+n)^9(\text{size}(A) + \text{size}(b) + \text{size}(c))^2)$. (5 Points)

Exercise 9.2:
Let G be a simple graph. Show that the following problem can be solved in time polynomial in $|V(G)|$.

$$\begin{array}{llcll}
\min & \displaystyle\sum_{e=\{v,w\}\in E(G)} x_{vw} & & & \\
s.d. & \displaystyle\sum_{w\in S} x_{vw} & \geq & \lceil \frac{1}{4}|S|^2 + \frac{1}{2}|S| \rceil & (v \in V(G), S \subseteq V(G)\setminus\{v\}) \\
 & x_{uw} & \leq & x_{uv} + x_{vw} & (u,v,w \in V(G)) \\
 & x_{vw} & \geq & 0 & (v \in V(G)) \\
 & x_{vv} & = & 0 & (v \in V(G))
\end{array}$$

(This is an LP-relaxation of the Optimal Linear Arrangement Problem: Find an ordering $\{v_1,\dots,v_{|V(G)|}\} = V(G)$ of the vertices such that $\sum_{\{v_i,v_j\}\in E(G)} |i-j|$ is minimum.) (5 Points)

Exercise 9.3:
Let $K \subseteq \mathbb{R}^n$ be a convex set with $B(0,r) \subseteq K \subseteq B(0,R)$ for some numbers $0 < r \leq \frac{R}{2}$. Assume that you are given an oracle with polynomial running time that computes an optimum solution in K for any linear objective function. Show that there is a separation oracle with polynomial running time for $K^* := \{y \in \mathbb{R}^n \mid y^t x \leq 1 \text{ for all } x \in K\}$. (6 Points)

Submission deadline: Thursday, December 14, 2017, before the lecture (in groups of 2 students).