Exercise Sheet 9

Exercise 9.1: Use the Ellipsoid Algorithm to show that a given feasible and bounded linear program \( \max \{ c^T x \mid Ax \leq b \} \) with \( A \in \mathbb{Q}^{m \times n} \), \( b \in \mathbb{Q}^m \) and \( c \in \mathbb{Q}^n \) can be solved in time \( O((m + n)^9(\text{size}(A) + \text{size}(b) + \text{size}(c))^2) \). (5 Points)

Exercise 9.2:
Let \( G \) be a simple graph. Show that the following problem can be solved in time polynomial in \( |V(G)| \).

\[
\begin{align*}
\min_{x} & \quad \sum_{\{v, w\} \in E(G)} x_{vw} \\
\text{s.t.} & \quad \sum_{w \in S} x_{vw} \geq \left[ \frac{1}{4} |S|^2 + \frac{1}{2} |S| \right] \quad (v \in V(G), S \subseteq V(G) \setminus \{v\}) \\
& \quad x_{uw} \leq x_{uv} + x_{vw} \quad (u, v, w \in V(G)) \\
& \quad x_{vw} \geq 0 \quad (v \in V(G)) \\
& \quad x_{vv} = 0 \quad (v \in V(G))
\end{align*}
\]
(This is an LP-relaxation of the Optimal Linear Arrangement Problem: Find an ordering \( \{v_1, \ldots, v_{|V(G)|}\} = V(G) \) of the vertices such that \( \sum_{\{v_i, v_j\} \in E(G)} |i-j| \) is minimum.) (5 Points)

Exercise 9.3:
Let \( K \subseteq \mathbb{R}^n \) be a convex set with \( B(0, r) \subseteq K \subseteq B(0, R) \) for some numbers \( 0 < r \leq \frac{R}{2} \). Assume that you are given an oracle with polynomial running time that computes an optimum solution in \( K \) for any linear objective function. Show that there is a separation oracle with polynomial running time for \( K^* := \{ y \in \mathbb{R}^n \mid y^T x \leq 1 \text{ for all } x \in K \} \). (6 Points)

Submission deadline: Thursday, December 14, 2017, before the lecture (in groups of 2 students).