Winter semester 2017/18 Research Institute for Discrete Mathematics
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Linear and Integer Optimization

Exercise Sheet 8

Exercise 8.1: Consider the **steepest edge pricing** applied to the network simplex algorithm.

1. Prove that the entering edge e_0 according to the steepest edge pricing rule can be computed in $\mathcal{O}(nm)$ time, where $n = |V(G)|$ and $m = |E(G)|$. (3 Points)
2. Show how e_0 can be computed in $\mathcal{O}((m+n)\alpha(m+n, n))$ time, where α is the inverse Ackermann function. (3* Points)

   (Hint: use the union-find data structure from EDM 16/17, english lecture notes can be found here:

   `http://jeffe.cs.illinois.edu/teaching/algorithms/notes/17-unionfind.pdf`)

Exercise 8.2: Let $P \subset \mathbb{R}^d$ be a finite set of points and let B be a ball containing P. Show: B is a minimum radius ball containing P if and only if the center of B lies in $\text{conv}(P \cap \partial B)$, where ∂B is the border of the ball. (6 Points)

Exercise 8.3:
Let

$$A := \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ s & -1 \end{pmatrix} \text{ and } b := \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}.$$

Use the Idealized Ellipsoid Algorithm with $R = 2$ to compute a feasible solution in $P = \{x \in \mathbb{R}^2 \mid Ax \leq b\}$ for $s = -1$ and for $s = -2$. (4 Points)

Submission deadline: Thursday, December 7, 2017, before the lecture (in groups of 2 students).