Winter semester 2017/18
Prof. Dr. S. Held
Dr. U. Brenner

Research Institute for Discrete Mathematics
University of Bonn

Linear and Integer Optimization

Exercise Sheet 6

Exercise 6.1: Consider the following LP with only one restricting equality:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} c_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} a_i x_i = b \qquad i = 1, \dots, n \\ & 0 \le x_i \le 1 \qquad i = 1, \dots, n. \end{aligned}$$

1. Provide a simple feasibility test for the problem.
2. Give an algorithm with running-time $\mathcal{O}(n \log n)$ that finds an optimum solution.

(5 Points)

Exercise 6.2: Let $G = (V, E)$ be a directed graph with edge capacities $u : E \to \mathbb{K}_+$ and let $s, t \in V$ be two special vertices. Furthermore, let

$$\mathcal{P} := \{P \subseteq E \mid P \text{ is the edge set of an } s\text{-}t\text{-path in } G\}.$$

Consider the following LP (P):

$$\begin{aligned} \max \quad & \sum_{P \in \mathcal{P}} y_P \\ \text{s.t.} \quad & \sum_{P \in \mathcal{P} \,:\, e \in P} y_P \le u(e) \qquad \text{for all } e \in E \\ & y_P \ge 0 \qquad\qquad\qquad\quad \text{for all } P \in \mathcal{P}. \end{aligned}$$

1. Determine the dual (D) of (P) and give graph theoretical interpretations of (D) and (P). (2 Points)
2. Find a class of graphs for which the number of paths $|\mathcal{P}|$ is not polynomially bounded by $|V| + |E|$. (2 Points)
3. Formulate an equivalent linear program to (P) for which the number of inequalities is polynomially bounded by $|V| + |E|$. (2 Points)

Exercise 6.3:
In the Network-Simplex, the fundamental circuit C of an edge $e \in E(G) \backslash T$ has to be computed in each iteration. If we have stored a pointer to the predecessor of v on the r-v-path in T for each vertex $v \in V(G)$, C can easily be determined in $\mathcal{O}(|V(G)|)$ time. On the other hand, $|V(G)| \gg |V(C)|$ holds for a lot of applications.

Show how the apex of C can be found

1. by traversing at most $2|V(C)|$ edges using the pointers to the predecessors and at most one additional memory-bit for each vertex. (2 Points)

2. by traversing at most $|V(C)|$ edges using the pointers to the predecessors and at most $\lceil \log n \rceil$ memory-bits for each vertex. (3 Points)

Submission deadline: Thursday, November 23, 2017, before the lecture (in groups of 2 students).