Winter semester 2017/18 Research Institute for Discrete Mathematics
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Linear and Integer Optimization

Exercise Sheet 4

Exercise 4.1: Let $P = P(A, b)$ be a polyhedron and F a minimal face of P. Prove that $Ax = Ay$ holds for all $x, y \in F$. (3 Points)

Exercise 4.2: Let C be a convex cone and $-C$ the cone $\{x : -x \in C\}$. We call $L = (C \cap -C)$ the **lineality space** of C.

a) Prove that $\bar{C} := C \cap L^{\perp}$, where $L^{\perp} = \{u : u^{\intercal}x = 0 \ \forall x \in L\}$, is a pointed cone and that C is the direct sum of its lineality space L and the pointed cone $\bar{C}$, i.e.
$$C = (C \cap L^{\perp}) \oplus L.$$
(2 Points)

b) Show that each polyhedron P has a decomposition $P = (Q + C) \oplus L$, where Q is a polytope, C a pointed cone and L a linear subspace. (3 Points)

Exercise 4.3: Given two extreme points a and b of a polyhedron P, we say that they are **adjacent** on P if the line segment between them forms an edge (i.e. a face of dimension 1) of P. Prove that a and b are adjacent on P, if and only if there exists a cost function c such that a and b are the only two extreme points of P minimizing $c^{\intercal}x$ over P. (3 Points)

Exercise 4.4: Let $H = (V, E)$ be a *hypergraph*, i.e. V is a finite set of vertices and $E \subseteq 2^V$. Furthermore, let $F \subseteq V$ and $x, y : F \to \mathbb{R}$. Provide an LP formulation for the following problem and dualize the LP:
Determine (an extension) $x, y : V \setminus F \to \mathbb{R}$ such that
$$\sum_{e \in E} (\max_{v \in e} x(v) - \min_{v \in e} x(v) + \max_{v \in e} y(v) - \min_{v \in e} y(v))$$
is minimized. (5 Points)

Remark: This is a relaxation of the placement problem in chip design. The vertices correspond to connected modules that must be placed minimizing the length of all interconnects (hyperedges). Vertices in F are preplaced. The problem becomes much harder when requiring disjointness of the modules.

Submission deadline: Thursday, November 9, 2017, before the lecture (in groups of 2 students).