Exercise 3.1:
(a) Prove the generalized Farkas lemma (Lemma 3.3): Let $A \in K^{m_1 \times n_1}, B \in K^{m_1 \times n_2}, C \in K^{m_2 \times n_1}, D \in K^{m_2 \times n_2}, a \in K^{m_1}$ and $b \in K^{m_2}$. Then exactly one of the following systems has a solution:
\[
\begin{align*}
Ax + By & \leq a \\
Cx + Dy & = b \\
x & \geq 0
\end{align*}
\]
\[
\begin{array}{ll}
\{ u^TA + v^TC & \geq 0 \} \ \lor \ \{ u^TB + v^TD & = 0 \} \\
\{ u & \geq 0 \} \\
\{ u^ta + v^tb & < 0 \}.
\end{array}
\]
(b) Let $(P)$ be a linear program of the form $\min\{c^Tx : Ax \leq b\}$. Prove that the dual of the dual is equivalent to $(P)$ (Lemma 3.11).

(2+2 points)

Exercise 3.2: Prove the following theorems of alternatives:
(a) $(\exists x : Ax \leq c, Ax \neq c)$
\[
\lor \left( \exists y : (A^Ty = 0, c^Ty = -1, y \geq 0) \lor (A^Ty = 0, c^Ty \leq 0, y > 0) \right).
\]
(3 Points)

(b) $(\exists x : Ax > 0, Cx \geq 0, Dx = 0)$
\[
\lor (\exists u, v, w : u, v \geq 0, u \neq 0, A^Tu + C^Tv + D^Tw = 0)
\]
(3 Points)

Exercise 3.3: Consider the following linear program $\min\{c^Tx : Ax = b\}$. Show that it either does not have a solution, is unbounded, or that all feasible solutions are optimal. Does this statement hold if we additionally require $x \geq 0$? (3 Points)
Exercise 3.4: Let $P$ be a polyhedron. Show that the problem of finding a largest ball that is contained in $P$ can be written as a linear program. 

(3 Points)

Submission deadline: Thursday, November 2, 2017, before the lecture (in groups of 2 students).