Winter semester 2017/18
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Linear and Integer Optimization

Exercise Sheet 1

Exercise 1.1:
A paper mill produces paper rolls of 3 m width. The customers order rolls with smaller widths and the mill has to cut the ordered rolls out of the 3 m wide rolls. For example, a 3 m wide roll may be cut into two 93 cm wide and a 108 cm wide roll, leaving an offcut of 6 cm.
The current order consists of

- 90 rolls of width 130 cm,
- 610 rolls of width 108 cm,
- 395 rolls of width 42 cm, and
- 211 rolls of width 93 cm.

Formulate a linear program that minimizes the number of produced 3 m rolls and allows a correct cutting of the ordered rolls.

(4 points)

Exercise 1.2: Specify necessary and sufficient conditions for the numbers $\alpha, \beta, \gamma \in \mathbb{K}$ so that the LP $\max\{x + y \ : \ \alpha x + \beta y \leq \gamma;\ x, y \geq 0\}$

- has an optimum solution;
- has a feasible solution;
- is unbounded.

(5 points)

Exercise 1.3:
Let G be a graph and consider the following LPs:

$$\begin{array}{llcll} \min & \sum_{v\in V(G)} x_v & & & \\ s.t. & x_v + x_w & \geq & 1 & (\{v,w\}\in E(G)) \\ & x_v & \geq & 0 & (v\in V(G)) \end{array} \tag{1}$$

$$\begin{array}{llcll} \max & \sum_{e\in E(G)} y_e & & & \\ s.t. & \sum_{e\in\delta(v)} y_e & \leq & 1 & (v\in V(G)) \\ & y_e & \geq & 0 & (e\in E(G)) \end{array} \tag{2}$$

1. Prove that both LPs are feasible and bounded.
2. Prove that the optimum solution value of the LP in (1) is an upper bound for the optimum solution value of the LP in (2).
3. Given an optimum x solution to (1), show how to compute an integral solution $x' \in \mathbb{Z}^{V(G)}$ satisfying the constraints of (1) and $\sum_{v\in V(G)} x'_v \leq 2\sum_{v\in V(G)} x_v$.

(1+3+2 points)

Exercise 1.4: The dimension of a non-empty set $X \subseteq \mathbb{K}^n$ is the number

$$\dim X := n - \max\{\operatorname{rank}(A) \;:\; A \text{ is an } n\times n \text{ matrix with } Ax = Ay\ \forall x,y\in X\}.$$

X is called **full-dimensional** if $\dim X = n$.
Prove: A polyhedron is full-dimensional if and only if there is a point in its interior.

(5 points)

Submission deadline: Thursday, October 19, 2017, before the lecture (in groups of 2 students).