Professor Dr. Stefan Hougardy
Jannik Silvanus

Exercise Set 12

Exercise 12.1. Let $G = (V, E)$ be a graph. We define

$$\mathcal{F} := \{X \subseteq V \mid X \text{ is covered by some matching}\},$$
$$\mathcal{F}^* := \{X \subseteq V \mid X \text{ is exposed by some maximum matching}\}.$$

(a) Show that $(V, \mathcal{F})$ and $(V, \mathcal{F}^*)$ are matroids.

(b) Show that $(V, \mathcal{F}^*)$ is the dual matroid of $(V, \mathcal{F})$.

(3+1 points)

Exercise 12.2. Let $n \in \mathbb{N}$. For what values of k is the uniform matroid of rank k on the set of n elements the graphic matroid of some simple graph?

(2 points)

Exercise 12.3. Let P be the convex hull of characteristic vectors of independent sets of a matroid $(E, \mathcal{F})$. Prove that $P \cap \{x \in \mathbb{R}^E \mid \sum_{e\in E} x_e = r(E)\}$ is the convex hull of characteristic vectors of bases of a matroid.

(2 points)

Exercise 12.4. Let $\mathcal{M}_1, \mathcal{M}_2$ be matroids on E. Let B be a maximal partitionable subset with respect to $\mathcal{M}_1$ and $\mathcal{M}_2^*$, and let J_1, J_2 be an associated partitioning. Furthermore, let B_2 be a basis of $\mathcal{M}_2^*$ with $J_2 \subseteq B_2$. Show that $B \setminus B_2$ is a common independent set of $\mathcal{M}_1$ and $\mathcal{M}_2$ of maximum cardinality.

(4 points)

Exercise 12.5. Let G be a connected graph and k an integer. Prove:
G has k edge-disjoint spanning trees if and only if there does not exist a partition of V into sets $V_0, \ldots, V_p$ such that $|E(V_0, \ldots, V_p)| < kp$, where $E(V_0, \ldots, V_p)$ denotes the set of edges with endpoints in different V_i.

(4 points)

Deadline: January 18th, before the lecture. The websites for lecture and exercises can be found at:

`http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html`

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.