Exercise Set 11

**Exercise 11.1.** Let $G$ be an undirected graph. Given a partition $(X_1, \ldots, X_k)$ of $V(G)$ we define $\delta(X_1, \ldots, X_k) := \delta(X_1) \cup \cdots \cup \delta(X_k)$ (so, in particular, if $\emptyset \neq X \subsetneq V(G)$ we have $\delta(X) = \delta(X, V(G) \setminus X)$). Consider the polytope

\[ R_G := \left\{ x : E(G) \to [0, 1] : \sum_{e \in E(G)} x(e) = |V(G)| - 1 \text{ and } \sum_{e \in \delta(X_1, \ldots, X_k)} x(e) \geq k - 1 \text{ for every partition } (X_1, \ldots, X_k) \text{ of } V(G) \right\} \]

(Compare with exercise 7.5.)

Show that $R_G$ is the spanning-tree polytope of $G$. (4 points)

**Exercise 11.2.** Consider the **Nearest Neighbor Heuristic** (NNH): Given an instance $(K_n, c)$ of the TSP, choose some $v_1 \in V(K_n)$. For $i = 2, \ldots, n$, choose $v_i \in V(K_n) \setminus \{v_1, \ldots, v_{i-1}\}$ such that $c(\{v_{i-1}, v_i\})$ is smallest possible. Return the tour given by the vertex sequence $(v_1, \ldots, v_n)$. Denote by $\text{opt}^{\text{NNH}}(K_n, c)$ the shortest possible length of any tour returned by the Nearest Neighbor Heuristic (i.e., taking the minimum over all possible choices within the algorithm), and by $\text{opt}(K_n, c)$ the length of an optimum tour. Show that the ratio $\text{opt}^{\text{NNH}}(K_n, c)/\text{opt}(K_n, c)$ can be arbitrarily large. (4 points)

**Exercise 11.3.** Given an instance $(K_n, c)$ of the TSP, denote by $HK(K_n, c)$ the Held-Karp lower bound and by $\text{opt}(K_n, c)$ the length of an optimum tour. Show that for instances of the Metric TSP the ratio $\text{opt}(K_n, c)/HK(K_n, c)$ can be arbitrarily close to $4/3$. (5 points)
Exercise 11.4. We consider the ANOTHER HAMILTONIAN CIRCUIT PROBLEM as defined in exercise 10.3.

Show that the ANOTHER HAMILTONIAN CIRCUIT PROBLEM, restricted to simple graphs $G$ with $|\delta(v)|$ odd for all $v \in V(G)$, is in $P$.

(3 points)

Deadline: January 11th, before the lecture. The websites for lecture and exercises can be found at:

http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de