Exercise 3.1. Let $G$ be a $k$-vertex-connected graph which has neither a perfect nor a near-perfect matching.

(i) Show that $\nu(G) \geq k$.

(ii) Show that $\tau(G) \leq 2 \cdot \nu(G) - k$.

(2+2 points)

Exercise 3.2. Given a bipartite graph $G$ and edge weights $c : E(G) \to \mathbb{R}_{\geq 0}$, we consider the iterations of the Hungarian method on $(G, c)$. Denote by $k$ the number of iterations, and for $1 \leq i \leq k$, denote by $G'_i$ the spanning subgraph of $G$ given by the edges satisfying $c\{x, y\} = w(x) + w(y)$ in the $i$-th iteration. Prove or disprove: For all $1 \leq i < k$, we have $E(G'_i) \subseteq E(G'_{i+1})$.

(4 points)

Exercise 3.3. Consider the Minimum Cost Edge Cover Problem: Given a graph $G$ with weights $c : E(G) \to \mathbb{R}_{\geq 0}$, find an edge cover $F \subseteq E(G)$ that minimizes $\sum_{e \in F} c(e)$. Show that the Minimum Cost Edge Cover Problem can be linearly reduced to the Minimum Weight Perfect Matching Problem.

(4 points)

Exercise 3.4. Consider the Shortest Even/Odd Path Problem: Given a graph $G$ with weights $c : E(G) \to \mathbb{R}_{\geq 0}$ and $s, t \in V(G)$, find an $s$-$t$-path $P$ of even/odd length in $G$ that minimizes $\sum_{e \in E(P)} c(e)$ among all $s$-$t$-paths of even/odd length in $G$. Show that both the even and the odd version can be linearly reduced to the Minimum Weight Perfect Matching Problem.

(4 points)

Deadline: November 2\textsuperscript{nd}, before the lecture. The websites for lecture and exercises can be found at

http://www.or.uni-bonn.de/lectures/ws17/co_exercises/exercises.html

In case of any questions feel free to contact me at silvanus@or.uni-bonn.de.