Winter term 2016/17
Prof. Dr. Stefan Hougardy
Tomás Silveira Salles

Research Institute for Discrete Mathematics
University of Bonn

Combinatorial Optimization
Exercise set 9

Exercise 9.1: Given a bipartite graph $G = (A\dot{\cup}B, E)$, we say that a B-*half-tour* in G is an edge-progression $v_1, e_1, v_2, \ldots, v_t, e_t, v_{t+1}$ with $v_1 = v_{t+1} \in A$ and such that every vertex of B appears precisely once in the sequence. (Vertices of A may appear any number of times or not at all, and edges may be repeated. And obviously $t = 2|B|$.)

Given $k, n \geq 1$ let $K_{k,n}$ be the complete bipartite graph with parts of sizes k and n, i.e., there is a bipartition $(A_{k,n}, B_{k,n})$ of $V(K_{k,n})$ with $|A_{k,n}| = k$, $|B_{k,n}| = n$ and every vertex of $A_{k,n}$ is connected to every vertex of $B_{k,n}$. Consider the Minimum Weight B-Half-Tour Problem: Given $k, n \geq 1$ and an edge-weight function $c : E(K_{k,n}) \to \mathbb{Q}_{\geq 0}$, find a $B_{k,n}$-half-tour in $K_{k,n}$ of minimum weight. Show that this problem can be solved in time that is polynomial with respect to n (although exponential with respect to k). (7 points)
Hint 1: Solve the "simpler" problem in which v_1 is pre-chosen (given as part of the input). Solving the original problem by solving k instances of this "simpler" version is fast enough.
Hint 2: Use exercise 8.3 considering the complete graph on $\{v_1\} \cup B$.

Exercise 9.2: Let $n \geq 4$ and $c : E(K_n) \to \mathbb{R}_{>0}$ be such that (K_n, c) is an instance of the Metric TSP, and let T be a tour on K_n. Show that there is a tour $T' \neq T$ such that

$$|c(T') - c(T)| \leq \frac{2}{n} \cdot c(T)$$

(4 points)

Exercise 9.3: Let $n \geq 3$ and $x : E(K_n) \to [0, 1]$ be such that it satisfies all degree constraints of the TSP but not all subtour elimination constraints. Show that there is a non-empty set $S \subsetneq V(K_n)$ such that

$$\sum_{e \in E(K_n[S])} x_e > |S| - 1$$

and $x_e < 1$ for all $e \in \delta(S)$. (4 points)

Exercise 9.4: Consider the Another Hamiltonian Circuit Problem: Given an undirected graph G and a Hamiltonian circuit C in G, decide whether there is any other Hamiltonian circuit in G. Show that this problem is NP-complete. (4 points)

Deadline: Thursday, January 12, 2017, before the lecture.
Note: Again, only hard copies this time. No submissions by e-mail!