Combinatorial Optimization
Exercise set 6

Exercise 6.1: Consider the **Minimum Weight Partial T-Join Problem**: Given an undirected graph $G$ with edge-weights $c : E(G) \rightarrow \mathbb{R}_{\geq 0}$ and disjoint sets $S, T \subseteq V(G)$, find a set $J \subseteq E(G)$ such that $|\delta(v) \cap J|$ is even for every $v \in S$ and odd for every $v \in T$ and such that $J$ has minimum weight among the sets with this property, or determine that no set with this property exists. Show that this problem can be linearly reduced to the **Minimum Weight T-Join Problem**. (4 points)

Exercise 6.2: Consider the **Minimum Weight Perfect Simple $b$-Matching Problem** (MWPS$_b$-MP): Given an undirected graph $G$, edge-weights $c : E(G) \rightarrow \mathbb{R}$ and a function $b : V(G) \rightarrow \mathbb{N}$, find a minimum weight perfect simple $b$-matching in $(G, c)$. (Note that the function $b$ is part of the input.) Using the fact that the **Minimum Weight Perfect Matching Problem** (MWPMP) can be solved in $O(|V(G)|^3)$-time, show that the MWPS$_b$-MP can be solved in $O(|E(G)|^3)$-time. (4 points)

Exercise 6.3: Consider the **Directed Chinese Postman Problem**: Given a strongly connected simple digraph $G$ with edge-weights $c : E(G) \rightarrow \mathbb{R}_{\geq 0}$, find a function $f : E(G) \rightarrow \mathbb{N} \setminus \{0\}$ such that if each edge $e \in E(G)$ is replaced by $f(e)$ copies of itself, the resulting graph is Eulerian, and such that $f$ minimizes $\sum\{f(e)c(e) : e \in E(G)\}$ among functions with this property. Show that this problem can be linearly reduced to the **Minimum Cost Integral Flow Problem** (i.e. the **Minimum Cost Flow Problem** with the additional requirement that the flow must be integral). (4 points)

Exercise 6.4: An *odd cover* for a graph $G$ is a set $F \subseteq E(G)$ such that if we successively contract in $G$ the elements of $F$ (and delete possible loops), the resulting graph is Eulerian. Consider the **Minimum Weight Odd Cover Problem**: Given an undirected graph $G$ with edge-weights $c : E(G) \rightarrow \mathbb{R}_{\geq 0}$, find an odd cover with minimum weight (or show that $G$ has no odd cover). Show that this problem can be solved in polynomial time. (4 points)

(Updated) Deadline: Thursday, December 8, 2016, before the lecture.