Winter term 2016/17
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Combinatorial Optimization
Exercise set 1

Exercise 1.1:

(i) Let G be a graph and suppose M_1 and M_2 are maximal matchings in G. Show that $|M_1| \leq 2 \cdot |M_2|$. (2 points)

(ii) Let G be a bipartite graph and suppose that for every non-empty $E' \subseteq E(G)$ we have $\tau(G - E') < \tau(G)$. Show that $E(G)$ is a matching in G. (2 points)

Exercise 1.2: Let G be a bipartite graph and let $V(G) = A \dot{\cup} B$ be a bipartition of G. If $A' \subseteq A$ and $B' \subseteq B$, and there are a matching $M_{A'}$ covering A' and a matching $M_{B'}$ covering B', show that there must be a matching covering $A' \cup B'$. (4 points)

Exercise 1.3: An edge of an undirected graph G is called *unmatchable* if it is not contained in any perfect matching of G. Show that the set of unmatchable edges of an undirected graph can be found in $O(n^3)$-time. (4 points)
Special deadline only for exercise 1.3: Thursday, November 3, 2016.

Exercise 1.4:

(i) Let G be a 3-regular undirected graph. Show that there is a matching in G covering at least $(7/8) \cdot |V(G)|$ vertices. (3 points)

(ii) Give an example to prove that the bound of the previous item is tight. (1 points)

Deadline: Thursday, October 27, 2016, before the lecture.