Exercise 1.1:

(i) Let \( G \) be a graph and suppose \( M_1 \) and \( M_2 \) are maximal matchings in \( G \). Show that \( |M_1| \leq 2 \cdot |M_2| \). (2 points)

(ii) Let \( G \) be a bipartite graph and suppose that for every non-empty \( E' \subseteq E(G) \) we have \( \tau(G - E') < \tau(G) \). Show that \( E(G) \) is a matching in \( G \). (2 points)

Exercise 1.2: Let \( G \) be a bipartite graph and let \( V(G) = A \cup B \) be a bipartition of \( G \). If \( A' \subseteq A \) and \( B' \subseteq B \), and there are a matching \( M_{A'} \) covering \( A' \) and a matching \( M_{B'} \) covering \( B' \), show that there must be a matching covering \( A' \cup B' \). (4 points)

Exercise 1.3: An edge of an undirected graph \( G \) is called unmatchable if it is not contained in any perfect matching of \( G \). Show that the set of unmatchable edges of an undirected graph can be found in \( O(n^3) \)-time. (4 points)

Special deadline only for exercise 1.3: Thursday, November 3, 2016.

Exercise 1.4:

(i) Let \( G \) be a 3-regular undirected graph. Show that there is a matching in \( G \) covering at least \( (7/8) \cdot |V(G)| \) vertices. (3 points)

(ii) Give an example to prove that the bound of the previous item is tight. (1 points)

Deadline: Thursday, October 27, 2016, before the lecture.