Research Institute for Discrete Mathematics
University of Bonn

Linear and Integer Optimization
Assignment Sheet 11

1. Assume that you are given a polynomial-time algorithm $\mathcal{A}$ that computes for any feasible and bounded linear program $\max\{c^t x \mid Ax \le b\}$ with $A \in \mathbb{Q}^{m\times n}$, $b \in \mathbb{Q}^m$ and $c \in \mathbb{Q}^n$ an optimum solution.

   (a) Show how to use $\mathcal{A}$ to compute in polynomial time an optimum solution x^* of $\max\{c^t x \mid Ax \le b\}$ such that x^* is contained in a minimal face of the polyhedron $\{x \in \mathbb{R}^n \mid Ax \le b\}$ (where we again assume that the problem is feasible and bounded).

   (b) Use $\mathcal{A}$ to find in polynomial running time an optimum solution of $\max\{c^t x \mid x \in P \cap \mathbb{Z}^n\}$ for a given non-empty integral polyhedron $P \subseteq \mathbb{R}^n$ for which the maximum is bounded. (4+5 points)

2. Prove that a polyhedral cone is rational if and only if it is generated by a finite number of integral vectors. Conclude that $C_I = C$ for any rational cone C. (6 points)

3. Let $a = (a_1, \dots, a_n) \in (\mathbb{N} \setminus \{0\})^n$ be a vector and β a rational number. Prove that $a^t x \le \beta$ is TDI if and only if the greatest common divisor of $a_1, \dots, a_n$ is 1 (4 points)

4. (a) Show that the systems

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

   and

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

   define the same polyhedron. Prove that the first system is TDI while the second one is not TDI.

   (b) Prove or disprove the following statement: If $Ax \le b$ (with $A \in \mathbb{Q}^{m\times n}$ and $b \in \mathbb{Q}^m$) is TDI and $\alpha \in \mathbb{Q}_{>0}$, then $\alpha Ax \le \alpha b$ is TDI. (3+3 points)

Due date: Tuesday, January 26, 2016, before the lecture.