Winter term 2015/16
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Linear and Integer Optimization
Assignment Sheet 10

1. Consider the following primal-dual pair of linear programs: $\max\{c^t x \mid Ax \leq b, x \geq 0\}$ and $\min\{b^t y \mid A^t y \geq c, y \geq 0\}$. Suppose that $\{x \in \mathbb{R}^n \mid Ax \leq b, x \geq 0\}$ is a non-empty polytope. Show that there is a feasible dual solution y with $y > 0$ and $A^t y > c$. (3 points)

2. Prove Lemma 59 in the lecture notes, i.e. show that if $y^{(k+1)}$ and $s^{(k+1)}$ are computed as described in the lecture, we have $y^{(k+1)} > 0$ and $s^{(k+1)} > 0$. (4 points)

3. Consider the following primal-dual pair of linear programs: (P): $\max\{c^t x \mid Ax \leq b\}$ and (D): $\min\{b^t y \mid A^t y = c, y \geq 0\}$ with $A \in \mathbb{R}^{m \times n}$. By strict complementary slackness, there is a partitioning $\{1, \dots, m\} = B \dot{\cup} N$ such that for $i \in B$ there is an optimum dual solution y^* with $y^*_i > 0$ and for $i \in N$ there is an optimum primal solution x^*, s^* with $s^*_i > 0$. Describe a linear program such that any optimum solution of it directly gives you the set B and N. (3 points)

4. Use the Interior Point Method to show that a given feasible and bounded linear program $\min\{b^t y \mid A^t y = c, y \geq 0\}$ with $A \in \mathbb{Q}^{m \times n}$, $b \in \mathbb{Q}^m$ and $c \in \mathbb{Q}^n$ can be solved in time $O((m + n)^{8.5}(\text{size}(A) + \text{size}(b) + \text{size}(c))^2)$. (6 points)

5. Give an example each of
   (a) a full-dimensional unbounded rational polyhedron P such that P_I is empty.
   (b) an unbounded polyhedron P such that P_I is non-empty and bounded.
   (c) a polyhedron P such that $P_I \neq \emptyset$ is not closed. (3+3+3 points)

Due date: Tuesday, January 19, 2016, before the lecture.