Winter term 2015/16 | Research Institute for Discrete Mathematics
Dr. U. Brenner | University of Bonn

Linear and Integer Optimization
Assignment Sheet 8

1. Let $A := \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ s & -1 \end{pmatrix}$ and $b := \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.
   Use the IDEALIZED ELLIPSOID ALGORITHM with $R = 2$ to compute a feasible solution in $P = \{x \in \mathbb{R}^2 \mid Ax \leq b\}$ for $s = -1$ and for $s = -2$. (5 points)

2. Use the ELLIPSOID ALGORITHM to show that a given feasible and bounded linear program $\max\{c^t x \mid Ax \leq b\}$ with $A \in \mathbb{Q}^{m\times n}$, $b \in \mathbb{Q}^m$ and $c \in \mathbb{Q}^n$ can be solved in time $O((m+n)^9(\text{size}(A) + \text{size}(b) + \text{size}(c))^2)$. (6 points)

3. Let $K \subseteq \mathbb{R}^n$ be a convex set with $rB^n \subseteq K \subseteq RB^n$ for some numbers $0 < r \leq \frac{R}{2}$.
   (a) Assume that there is a polynomial-time algorithm $\mathcal{A}$ that for a given $x \in \mathbb{R}^n$ either asserts $x \in K$ or returns a vector $v \in \mathbb{R}^n$ with $\max\{v^t z \mid z \in K\} \leq 1$ and $1 < v^t x$. Show that there is an algorithm for the following problem with running time polynomial in n, $\ln(\frac{1}{r})$, $\ln(R)$, $\ln(\frac{1}{\epsilon})$, and size(c): Given a vector $c \in \mathbb{R}^n$ with $\|c\| = 1$ and a value $\epsilon > 0$ compute an $\in K$ with $c^t x \geq \max\{c^t z \mid z \in K\} - \epsilon$.
   (b) Assume that you are given an oracle with polynomial running time that computes an optimum solution in K for any linear objective function. Show that there is a separation oracle with polynomial running time for $K^* := \{y \in \mathbb{R}^n \mid y^t x \leq 1 \text{ for all } x \in K\}$. (6+2 points)

4. Let $P \subset \mathbb{R}^d$ be a finite set of points and let B be a ball containing P. Show: B is a minimum radius ball containing P if and only if the center of B lies in $\text{conv}(P \cap \partial B)$, where ∂B is the border of the ball. (6 points)

Due date: Tuesday, December 22, 2015, before the lecture.