

Linear and Integer Optimization

Assignment Sheet 5

1. Use the **SIMPLEX ALGORITHM** to solve the following linear programs:

(a)

$$\begin{array}{ll}\max & 2x_2 \\ \text{s.t.} & x_1 - x_2 \leq 4 \\ & -x_1 + x_2 \leq 1 \\ & x_1, x_2 \geq 0\end{array}$$

(b)

$$\begin{array}{ll}\max & 5x_1 + 3x_2 \\ \text{s.t.} & 4x_1 + 2x_2 \leq 12 \\ & 4x_1 + x_2 \leq 10 \\ & x_1 + x_2 \leq 4 \\ & x_1, x_2 \geq 0\end{array}$$

(c)

$$\begin{array}{ll}\min & 5x_1 - x_2 \\ \text{s.t.} & x_1 - 3x_2 \leq 1 \\ & x_1 - 4x_2 \leq 3 \\ & x_1, x_2 \geq 0\end{array}$$

(d)

$$\begin{array}{ll}\min & -x_1 - 2x_2 \\ \text{s.t.} & 2x_1 + x_2 \leq 5 \\ & -x_1 - x_2 \geq -3 \\ & x_1, x_2 \geq 0\end{array}$$

Show all intermediate simplex tableaus and describe why you can choose a specific variable to enter or leave the basis. If there is an optimum solution, also give its value.
(2+2+2+2 points)

2. Consider a linear program $\max\{c^t x \mid Ax = b, x \geq 0\}$. Let B be a feasible basis with basic solution x^* and reduced cost vector $r \leq 0$ (so x^* is an optimum solution). Let $I = \{j \in N \mid r_j = 0\}$.

- (a) Prove that x^* is the unique optimum solution if $I = \emptyset$.
 (b) Assume that $I \neq \emptyset$. Prove that in this case x^* is the unique optimum solution if and only if the following linear program has the optimum solution value 0:

$$\begin{aligned} \max \quad & \sum_{i \in I} x_i \\ \text{s.t.} \quad & Ax = b \\ & x_i = 0 \quad \text{for } i \in N \setminus I \\ & x_i \geq 0 \quad \text{for } i \in B \cup I \end{aligned}$$

(5 points)

3. Show that the MAXIMUM-FLOW PROBLEM can be considered as a special case of the MINIMUM-COST FLOW PROBLEM. (2 points)

Due date for the first 3 exercises: Thursday, December 3, 2015, before the lecture.

4. Programming Exercise 2

Implement the SIMPLEX ALGORITHM to solve linear programs of the type $\max\{c^t x \mid Ax \leq b, x \geq 0\}$. Your program should decide if an instance is infeasible, unbounded or can be solved optimally. If an instance can be solved optimally, your program should output an optimum solution and the corresponding value of the objective function. You may choose a pivot rule by yourself. The input is specified as in the first programming exercise on Assignment Sheet 2. Instances and the input reader can be reused. (10 Points)

Due date for the programming exercise: Tuesday, December 15, 2015, before the lecture.
 Please send your solution by e-mail to your tutor.