Winter term 2015/16
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Linear and Integer Optimization
Assignment Sheet 4

1. Two sets $X, Y \subseteq \mathbb{R}^n$ are called *strictly separable*, if there is a hyperplane $\{x \in \mathbb{R}^n \mid a^t x = b\}$ such that $a^t x < b$ for all $x \in X$ and $a^t y > b$ for all $y \in Y$. Prove or disprove the following statement: Any two disjoint closed convex sets are strictly separable. (4 points)

2. For a set $X \subseteq \mathbb{R}^n$ let $X^* := \{y \in \mathbb{R}^n \mid y^t x \leq 1 \text{ for all } x \in X\}$. Let $P \subseteq \mathbb{R}^n$ be a polyhedron with $0 \in P$. Prove the following statements:

   (a) P^* is a polyhedron.

   (b) $(P^*)^* = P$.

   (c) P has dimension n if and only if P^* is pointed.

   (d) 0 is an interior point of P if and only if P^* is bounded. (3+3+3+3 points)

3. Consider the linear program (P)

$$\begin{array}{rrcl} & \max c^t x & & \\ \text{s.t.} & Ax & \leq & b \\ & x & \geq & 0 \end{array}$$

   Let $\tilde{x}$ be a non-degenerated feasible basic solution of (P) with value $\delta = c^t \tilde{x}$, and let $\tilde{y} = (\tilde{y}_1, \ldots, \tilde{y}_m)$ be an optimum solution of its dual. Show that there is a positive ϵ such that for any vector $p = (p_1, \ldots, p_m)$ with $p_i \in [0, \epsilon]$ $(i = 1, \ldots, m)$ the modified linear program (P')

$$\begin{array}{rrcl} & \max c^t x & & \\ \text{s.t.} & Ax & \leq & b + p \\ & x & \geq & 0 \end{array}$$

   has an optimum solution of value $\delta + \tilde{y}^t p$. (5 points)

4. Prove or disprove the following statement: If $X, Y \subseteq \mathbb{R}^n$ are polyhedra, then $X + Y$ is a polyhedron. (4 points)

Due date: Tuesday, November 24, 2015, before the lecture.

An announcement of the student council of mathematics: The student council of mathematics will organize the math party on 26.11. in N8schicht. The presale will be held on Mon 23.11., Tue 24.11. and Wed 25.11. in front of the mensa Poppelsdorf. Further information can be found at `fsmath.uni-bonn.de`