1. Let $X \subseteq \mathbb{R}^n$ and $y \in \text{conv}(X)$. Prove that there are vectors $x_1, \ldots, x_k \in X$ with $k \leq n + 1$ and $y \in \text{conv}\{x_1, \ldots, x_k\}$. (5 points)

2. Let $P$ be a polyhedron with $\text{dim}(P) = d$ and $F$ a face of $P$ with $\text{dim}(F) = k \in \{0, \ldots, d-1\}$. Show that there are faces $F_{k+1}, F_{k+2}, \ldots, F_{d-1}$ of $P$ with
   
   i) $F \subseteq F_{k+1} \subseteq F_{k+2} \subseteq \cdots \subseteq F_{d-1} \subseteq P$,
   
   ii) $\text{dim}(F_{k+i}) = k + i$ for $i \in \{1, \ldots, d - k - 1\}$. (5 points)

3. For $n \in \mathbb{N} \setminus \{0\}$ and a subset $X \subseteq \mathbb{R}$ let

   
   $$M_X = \left\{ A = (a_{ij})_{i=1,...,n} \mid a_{i0j0} \in X, \sum_{i=1}^{n} a_{i0j} = 1, \sum_{j=1}^{n} a_{i0j} = 1 \text{ (for } i_0, j_0 \in \{1, \ldots, n\}\right\}.$$ 

   Show that an $n \times n$-matrix $A$ is in $M_{\mathbb{R}_{\geq 0}}$ if and only if it is a convex combination of matrices in $M_{\{0,1\}}$. (6 points)

4. In the job assignment problem, $n$ jobs with execution times $t_1, \ldots, t_n \in \mathbb{R}_{\geq 0}$ need to be processed by $m$ workers. For each job $i$ we are given by $S_i \subseteq \{1, \ldots, m\}$ the set of workers that are qualified to perform job $i$. It is possible for several workers to process the same job in parallel to speed up the process but one worker can only process one job at a time.

   (a) Formulate an LP minimizing the makespan for processing all jobs (the time until the last worker finishes).

   (b) Dualize this LP.

   (c) Develop a simple polynomial time algorithm for $n = 2$ that finds an optimal solution (for the primal problem) and prove its correctness. (3+3+3 points)

Due date: Tuesday, November 17, 2015, before the lecture.