Winter term 2015/16
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Combinatorial Optimization

Exercise Sheet 8

Exercise 8.1: Given an undirected graph, an *odd cycle cover* is defined to be a subset of the edges containing at least one edge of each odd circuit. Show how to find in polynomial time a minimum weight odd cycle cover in a planar graph with nonnegative weights on the edges. Can you also solve the problem for general weights?

(4 Points)

Exercise 8.2: Let G be an undirected graph and $T \subseteq V(G)$ with $|T| = 2k$ even. Prove that the minimum cardinality of a T-cut in G equals the maximum of $\min_{i=1}^{k} \lambda_{s_i,t_i}$ over all pairings $T = \{s_1, t_1, \ldots, s_k, t_k\}$, where $\lambda_{s,t}$ denotes the maximum number of pairwise edge-disjoint s-t-paths.
Hint: Use the Padberg-Rao-Theorem.

(4 Points)

Exercise 8.3: The Directed Chinese Postman Problem can be formulated as follows: Given a strongly connected simple digraph G with weights $c : E(G) \to \mathbb{R}_+$, find $f : E(G) \to \mathbb{N}$ such that the graph which contains $f(e)$ copies of each edge $e \in E(G)$ is Eulerian and $\sum_{e \in E(G)} c(e)f(e)$ is minimum. Show how to solve this problem in polynomial time by reducing it to a Minimum Cost Flow Problem.

(4 Points)

Exercise 8.4: Let G be an undirected graph and $b_1, b_2 : V(G) \to \mathbb{N}$. Describe the convex hull of functions $f : E(G) \to \mathbb{Z}_+$ with $b_1(v) \le \sum_{e \in \delta(v)} f(e) \le b_2(v)$.
Hint: For $X, Y \subseteq V(G)$ with $X \cap Y = \emptyset$ consider the constraint

$$\sum_{e \in E(G[X])} f(e) - \sum_{e \in E(G[Y]) \cup E(Y,Z)} f(e) \le \left\lfloor \frac{1}{2} \left(\sum_{x \in X} b_2(x) - \sum_{y \in Y} b_1(y) \right) \right\rfloor ,$$

where $Z := V(G) \setminus (X \cup Y)$. Use the b-matching polytope. (4 Points)

Deadline: Tuesday, December 22, 2015, **before** the lecture.
Information: Submissions by groups of up to **three** students are allowed.