Combinatorial Optimization

Exercise Sheet 8

Exercise 8.1: Given an undirected graph, an odd cycle cover is defined to be a subset of the edges containing at least one edge of each odd circuit. Show how to find in polynomial time a minimum weight odd cycle cover in a planar graph with nonnegative weights on the edges. Can you also solve the problem for general weights?

(4 Points)

Exercise 8.2: Let $G$ be an undirected graph and $T \subseteq V(G)$ with $|T| = 2k$ even. Prove that the minimum cardinality of a $T$-cut in $G$ equals the maximum of $\min_{i=1}^{k} \lambda_{s_i,t_i}$ over all pairings $T = \{s_1,t_1,\ldots,s_k,t_k\}$, where $\lambda_{s,t}$ denotes the maximum number of pairwise edge-disjoint $s$-$t$-paths.

Hint: Use the Padberg-Rao-Theorem.

(4 Points)

Exercise 8.3: The Directed Chinese Postman Problem can be formulated as follows: Given a strongly connected simple digraph $G$ with weights $c : E(G) \to \mathbb{R}_+$, find $f : E(G) \to \mathbb{N}$ such that the graph which contains $f(e)$ copies of each edge $e \in E(G)$ is Eulerian and $\sum_{e \in E(G)} c(e)f(e)$ is minimum. Show how to solve this problem in polynomial time by reducing it to a Minimum Cost Flow Problem.

(4 Points)

Exercise 8.4: Let $G$ be an undirected graph and $b_1,b_2 : V(G) \to \mathbb{N}$. Describe the convex hull of functions $f : E(G) \to \mathbb{Z}_+$ with $b_1(v) \leq \sum_{e \in \delta(v)} f(e) \leq b_2(v)$.

Hint: For $X,Y \subseteq V(G)$ with $X \cap Y = \emptyset$ consider the constraint

$$\sum_{e \in E(G[X])} f(e) - \sum_{e \in E(G[Y]) \cup E(Y,Z)} f(e) \leq \frac{1}{2} \left( \sum_{x \in X} b_2(x) - \sum_{y \in Y} b_1(y) \right),$$

where $Z := V(G) \setminus (X \cup Y)$. Use the b-matching polytope.

(4 Points)

Deadline: Tuesday, December 22, 2015, before the lecture.

Information: Submissions by groups of up to three students are allowed.