Winter term 2015/16
Prof. Dr. Stephan Held
Prof. Dr. Jens Vygen
Pascal Cremer

Research Institute for Discrete Mathematics
University of Bonn

Combinatorial Optimization

Exercise Sheet 7

Exercise 7.1: Let G be a graph, $T \subseteq V(G)$ with $|T|$ even, and $F \subseteq E(G)$. A subset $C \subseteq E(G)$ is called a T-*cut* if $C = \delta(U)$ for some $U \subseteq V(G)$ with $|U \cap T|$ odd. Prove:

(i) F has nonempty intersection with every T-join if and only if F contains a T-cut.

(ii) F has nonempty intersection with every T-cut if and only if F contains a T-join.

(4 Points)

Exercise 7.2: Let G be a graph with edge weights $c : E(G) \to \mathbb{R}_+$. A set $F \subseteq E(G)$ is called *odd cover* if the graph which results from G by successively contracting each $e \in F$ is Eulerian. Show that it is possible in polynomial time to find an odd cover F that minimizes $c(F)$ or to decide that none exists. We use the notation $c(F) := \sum_{e \in F} c(e)$ for edge sets $F \subset E(G)$.

(4 Points)

Exercise 7.3: Consider the MAXIMUM WEIGHT CUT PROBLEM in planar graphs: Given an undirected planar graph G with weights $c : E(G) \to \mathbb{R}_+$, we look for a maximum weight cut in G. How can this problem be solved in polynomial time?
Hint: Use Exercise 7.2 and the following fact: A connected undirected graph is bipartite if and only if its planar dual is Eulerian.

Note: For general graphs this problem is NP-hard even for unit weights.

(4 Points)

Continued on next page.

Exercise 7.4: Let G be a planar 2-connected graph with fixed embedding, let C be the circuit bounding the outer face, and let T be an even cardinality subset of $V(C)$. Prove that the minimum cardinality of a T-join equals the maximum number of pairwise edge-disjoint T-cuts.
Hint: Color the edges of C red and blue such that, when traversing C, colors change precisely at the vertices in T. Consider the planar dual graph, split the vertex representing the outer face into a red and a blue vertex, and apply Menger's Theorem.
(4 Points)

Deadline: Tuesday, December 15, 2015, **before** the lecture.
Information: Submissions by groups of up to **three** students are allowed.